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Performance analysis of noncoherent DPSK with various diversity combining techniques over a Rician fading channel

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THESIS

PERFORMANCE ANALYSIS OF NONCOHERENT DPSK WITH VARIOUS DIVERSITY COMBINING TECHNIQUES OVER A RICIAN FADING CHANNEL

by

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I. INTRODUCTION

To improve the performance in communication systems, especially in a fading or jamming environment, diversity has long been used as an effective technique by reducing the effects of channel fading. Fading is a destructive addition of the signal components, arriving at the receiver by different propagation paths. When the communications signal is subject to fading, the error probability increases due to the amplitude variations in the received signal components which are the result of phase variations. The phase variation is caused by the time-varying multipath characteristics of the channel. The phase differences in the received signal components cause a noncoherent combining loss at the receiver. To minimize performance losses one could require to increase the transmitter power but in most cases this is undesirable. To reduce the degradation effects of channel fading, one of several well-known diversity techniques can be applied either at the transmitter or at the receiver.

Channel diversity is a powerful technique that provides relatively low cost link improvement. The basic concept is to provide L independent replicas of the transmitted signal over independent possibly fading channels to the receiver. The probability that all signals will fade simultaneously is small. One of the transmission paths may be subject to a deep fade while some other paths may have strong signal. In such an environment, by using more than one signal, both the average and the instantaneous signal-to-noise (SNR) at the receiver can be improved. Time diversity of order L is the redundancy where each information bit is transmitted L times or once over L channels. Each one of the L

channels is a so called a diversity branch which in our case is subject to independent Rician fading.

In general, the diversity methods can be classified as frequency, time, space (antenna), angle or polarization diversity. When using frequency diversity, L different frequencies carry identical signals. The separation between the carriers must be sufficiently large to allow a separation that equals or exceeds the coherence bandwidth $(\Delta f)_c$ of the channel. Here coherence bandwidth is defined as the frequency range over which the signal can pass without distortion. A frequency non-selective channel provides a coherence bandwidth that is larger than the signal bandwidth. Another diversity method is the time diversity in which L information-bearing signals are transmitted in L different time slots. The separation between successive time slots must equal or exceed the coherence time $(\Delta t)_c$ of the channel. Here coherence time is defined as

$$(\Delta t)_c = \frac{1}{B_d} , \tag{1}$$

where B_d is the Doppler spread of the channel. A slowly fading channel provides a larger coherence time.

Space diversity (antenna diversity), which is widely used because of its simple and economic implementation, uses a single transmitting antenna and several receiving antennas. The receiving antennas must be placed sufficiently apart to have significantly different propagation delays to each antenna. Generally a 10-wavelength separation between any two antennas is considered to be sufficient. Angle diversity (direction diversity) requires a set of directive antennas where each one responds independently to a

wave to produce an uncorrelated faded signal. In Polarization diversity, only two diversity branches are available. This is achieved when signals that are transmitted use two orthogonal polarizations.

The advantage of frequency and time diversity compared with space, angle and polarization diversity techniques is that the number of transmitting and receiving antennas does not need to be increased. On the other hand, they require a wider bandwidth.

To obtain a performance improvement, the information from the diversity branches must be combined efficiently. For this purpose a number of powerful and robust diversity combining methods is examined to combine uncorrelated fading signals obtained from the diversity branches.

Diversity combining can be accomplished before or after the signal detection. If the diversity combining is applied before the signal detection, it is called pre-detection combining. Conversely, if it is done after the signal detection, it is called post-detection combining.

In reference 1, equal gain combining and different orders of selection combining techniques (SC, SC-2 and SC-3) are investigated for noncoherent Differential Phase Shift Keying (DPSK) in a Rayleigh fading channel. The bit-error rate performance of the diversity combining methods shows that combining the two or three largest signals (SC-2 or SC-3) offers significant performance improvement over just selecting the largest signal component and the performance improves as L increases.

In this thesis, Equal Gain Combining (EGC), Selection Combining (SC) and Post Detection Selection Combining (PDSC) techniques are analyzed for the noncoherent DPSK receiver [2] in a Rician fading channel.

The communication channel is modeled as a frequency-non-selective, slowly-fading Rician channel and is assumed to be perturbed by additive white Gaussian noise.

A. RICIAN FADING CHANNEL

For the receiver investigated in this thesis, the received signal amplitude is assumed to be a Rician random variable. As the signal travels to the receiver, it is reflected by the surrounding objects which causes indirect (diffuse) signal components. The receiver may pick up the direct signal component from the line-of-sight path and an indirect (diffuse) signal component from a reflected path. The indirect signal component is also referred to as multipath. The multipath components destructively interact and reduce the performance of the receiver. This phenomenon is called multipath fading.

If the line-of-sight component is not zero, then the received signal amplitude is modeled as a Rician random variable and the channel is referred to as a Rician fading channel.

For a Rician faded signal, the probability density function of the received signal amplitude is [3]

$$f_R(r) = \left\{ \begin{array}{ll} \dfrac{r}{\sigma^2} \exp\left(-\dfrac{(r^2+\alpha^2)}{2\sigma^2} \right) I_0\left(\dfrac{\alpha r}{\sigma^2} \right) & , \; r > 0 \\ 0 & , \; otherwise \end{array} \right\} ; \qquad (2)$$

where α^2 is the average power of the unfaded (direct) signal component, $2\sigma^2$ is the average power of the faded (diffuse) signal component and $I_o(\bullet)$ is the zeroth-order modified Bessel function of the first kind. The total power of the received signal is the sum of the power of the direct and diffuse signal components, $\alpha^2 + 2\sigma^2$. The ratio of the direct signal component power to the diffuse signal component power is

$$\gamma = \frac{\alpha^2}{2\sigma^2} = \frac{direct\ component\ power}{diffuse\ component\ power}. \tag{3}$$

Here γ characterizes the strength of the fading channel. We assume that each diversity reception fades independently and the channel can be modelled as a slowly fading, frequency non-selective Rician fading channel.

When the direct signal component is zero, that is $\alpha = 0$ and $\gamma = 0$, the received signal amplitude is modeled as a Rayleigh random variable and the channel is called a Rayleigh fading channel. The probability density function of the received signal amplitude in a Rayleigh fading channel is [3]

$$f_R(r) = \left\{ \begin{array}{ll} \frac{r}{\sigma^2}\exp(-\frac{r^2}{2\sigma^2}), & r \geq 0 \\ 0 & , otherwise \end{array} \right\}. \tag{4}$$

B. NONCOHERENT DPSK RECEIVER

The block diagram of the noncoherent DPSK receiver, as used in this thesis, is shown in Fig. 1 [2].

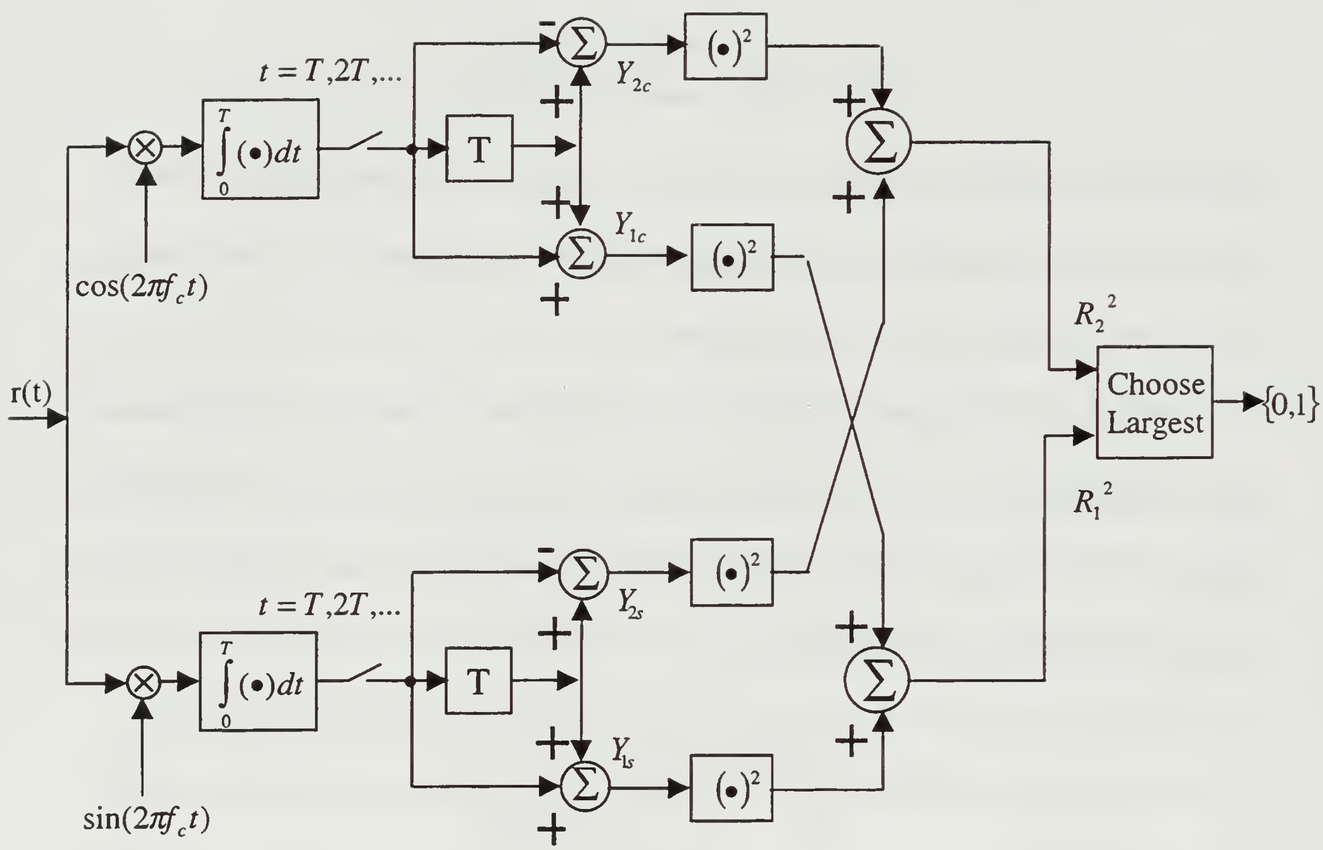

Figure 1. Noncoherent DPSK receiver

DPSK is an efficient modulation method in fading channels when it is difficult tosynchronize the phase. It requires phase stability over only two consecutive signaling intervals.

In DPSK, when the data bit $b_i = 0$ is transmitted, the waveform $v^{(1)}(t)$ for two consecutive differentially encoded bits $c_{i-1}c_i$ is [2]

$$\begin{aligned} v^{(1)}(t) &= \pm A\, p_T(t-(i-1)T)\cos(2\pi f_c t+\theta_0) \pm A\, p_T(t-iT)\cos(2\pi f_c t+\theta_0) \\ &= [\pm A\, p_T(t-(i-1)T) \pm A\, p_T(t-iT)]\cos(2\pi f_c t+\theta_0), \end{aligned} \tag{5}$$

where $p_T(\bullet)$ is a signal pulse of duration of one bit interval given by

$$p_T(t-iT) = \begin{cases} 1\ , & iT < t \leq (i+1)T \\ 0\ , & otherwise \end{cases}. \tag{6}$$

θ_0 is the signal phase, f_c is the carrier frequency, A is the amplitude and $\pm$ denotes the polarity of the two consecutive bits.

When the data bit $b_i = 1$ is transmitted, there is a phase change of π radians between two consecutive bits. Hence, the waveform $v^{(2)}(t)$ is

$$\begin{aligned} v^{(2)}(t) &= \pm A\, p_T(t-(i-1)T)\cos(2\pi f_c t+\theta_0) \mp A\, p_T(t-iT)\cos(2\pi f_c t+\theta_0) \\ &= [\pm A\, p_T(t-(i-1)T) \mp A\, p_T(t-iT)]\cos(2\pi f_c t+\theta_0). \end{aligned} \tag{7}$$

If we assume $b_1 = 0$ is transmitted, the received signal with white Gaussian noise n(t) is of the form

$$r(t) = [\pm A\, p_T(t) \pm A\, p_T(t-T)]\cos(2\pi f_c t+\theta_0) + n(t). \tag{8}$$

The in phase outputs of the receiver are given by

$$Y_{1c} = \left(\pm \frac{AT}{2}\cos\theta_0 + N_{1c}' \right) + \left(\pm \frac{AT}{2}\cos\theta_0 + N_{1c}'' \right) = \pm AT\cos\theta_0 + N_{1c} \tag{9}$$

and

$$Y_{2c} = \left(\pm \frac{AT}{2}\cos\theta_0 + N_{1c}' \right) - \left(\pm \frac{AT}{2}\cos\theta_0 + N_{1c}'' \right) = N_{2c}, \tag{10}$$

where

$$N_{1c} = N_{1c}' + N_{1c}'' \quad \text{and} \quad N_{2c} = N_{1c}' - N_{1c}'' . \tag{11}$$

N_{1c}' and N_{1c}'' are described as

$$N_{1c}' = \int_0^T n(t)\cos(2\pi f_c t)\,dt \tag{12}$$

and

$$N_{1c}'' = \int_T^{2T} n(t)\cos(2\pi f_c t)\,dt . \tag{13}$$

The quadrature phase outputs are given by

$$Y_{1s} = \left(\pm \frac{AT}{2}(-\sin\theta_0) + N_{1s}' \right) + \left(\pm \frac{AT}{2}(-\sin\theta_0) + N_{1s}'' \right) = \mp AT\sin\theta_0 + N_{1s} \tag{14}$$

and

$$Y_{2s} = \left(\pm \frac{AT}{2}(-\sin\theta_0) + N_{1s}' \right) - \left(\pm \frac{AT}{2}(-\sin\theta_0) + N_{1s}'' \right) = N_{2s} , \tag{15}$$

where

$$N_{1s} = N_{1s}' + N_{1s}'' \quad \text{and} \quad N_{2s} = N_{1s}' - N_{1s}'' . \tag{16}$$

N_{1s}' and N_{1s}'' are described as

$$N_{1s}' = \int_0^T n(t)\sin(2\pi f_c t\,)dt \tag{17}$$

and

$$N_{1s}'' = \int_T^{2T} n(t)\sin(2\pi f_c t)dt\,. \tag{18}$$

The noise n(t) is modeled as additive white Gaussian noise with a two sided power spectral density $\frac{N_0}{2}$. The noise is assumed to be unaffected by the fading channel. Therefore N_{1c}, N_{2c}, N_{1s} and N_{2s} are independent, identically distributed (i.i.d.), zero mean Gaussian random variables with variance $\frac{N_0 T}{2}$.

If we assume that $b_i = 0$ is transmitted, then for the lower branch (Fig.1.) which is the signal branch, the outputs are given by

$$\begin{aligned} Y_{1c} &= \pm AT\cos\theta + N_{1c} \\ Y_{1s} &= \mp AT\sin\,\theta + N_{1s}\,. \end{aligned} \tag{19}$$

The upper branch, which is the non-signal branch, is described by

$$\begin{aligned} Y_{2c} &= N_{2c} \\ Y_{2s} &= N_{2s}\,. \end{aligned} \tag{20}$$

The decision variables are defined as

$$R_1^{\,2} = Y_{1c}^{\;2} + Y_{1s}^{\;2} = (\pm\,AT\cos\theta + N_{1c})^2 + (\mp\,AT\sin\theta + N_{1s})^2 \tag{21}$$

for the signal branch and

$$R_2^{\,2} = Y_{2c}^{\;2} + Y_{2s}^{\;2} = N_{2c}^{\;2} + N_{2s}^{\;2} \tag{22}$$

for the non-signal branch.

In chapter II, III and IV the bit error probability expressions of the EGC, the SC and the PDSC techniques are analytically obtained and the numerical results are presented in chapter V.

II. EQUAL GAIN COMBINING

Equal Gain Combining (EGC) is a technique in which the information from all available diversity branches (paths) is added with equal weights. In other words, the L-branch equal gain combiner outputs V_1 which is the decision variable for the signal branch, and V_2 which is the decision variable for the non-signal branch, are the sum of L independent random variables,

$$\begin{aligned} V_1 &= R_{11}^2 + R_{12}^2 + R_{13}^2 + \ldots + R_{1L}^2 \\ V_2 &= R_{21}^2 + R_{22}^2 + R_{23}^2 + \ldots + R_{2L}^2, \end{aligned} \tag{23}$$

where R_{1i}^2 and R_{2i}^2, $i = (1,2,\ldots L)$, are the amplitudes of the signal received by the i^{th} channel. The receiver for EGC is shown in Fig. 2.

This method has been commonly used by receivers that use noncoherent demodulation and is considered to be equivalent to Maximum Ratio Combining (MRC) for coherent demodulation [3].

Due to the "noncoherent combining loss" effect, the EGC method is not optimal. In particular, increasing the number of diversity channels (L) does not necessarily improve the performance.

A. PROBABILITY DENSITY FUNCTION OF THE DECISION VARIABLES

In the noncoherent DPSK receiver as shown in Fig. 2, the decision variable V_1 is the sum of the squares of 2L Gaussian random variables. If we assume that the data bit

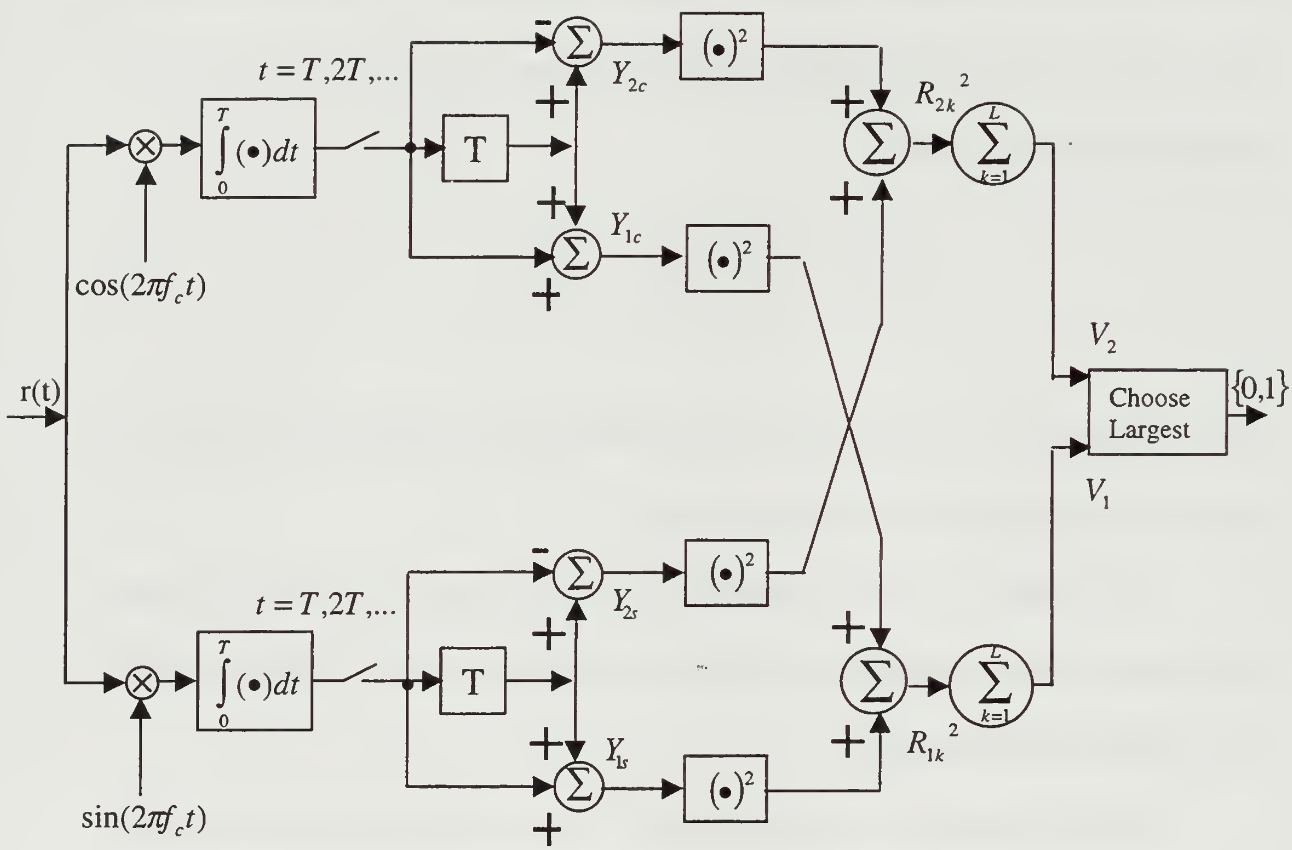

Figure 2. Noncoherent DPSK receiver for Equal Gain Combining (EGC) with L^{th} order diversity

$b_i = 0$ is transmitted, then the output for the signal present branch is

$$V_1 = \sum_{k=1}^{L} R_{1k}^{\ 2}. \tag{24}$$

For convenience, we normalize (21) as follows;

$$R_1^{\ 2} = \left[\sqrt{\frac{T}{2}}\left(\pm\sqrt{2T}A\cos\theta + \frac{\sqrt{2}N_{1c}}{\sqrt{T}}\right)\right]^2 + \left[\sqrt{\frac{T}{2}}\left(\mp\sqrt{2T}A\sin\theta + \frac{\sqrt{2}N_{1s}}{\sqrt{T}}\right)\right]^2$$

$$= \frac{T}{2}\left(\pm\sqrt{2T}A\cos\theta + \frac{\sqrt{2}N_{1c}}{\sqrt{T}}\right)^2 + \frac{T}{2}\left(\mp\sqrt{2T}A\sin\theta + \frac{\sqrt{2}N_{1s}}{\sqrt{T}}\right)^2. \tag{25}$$

By setting $R_{1k}^{\ 2} = \frac{2}{T}R_1^{\ 2}$, $n_{1c_k} = \sqrt{\frac{2}{T}}N_{1c}$, $n_{1s_k} = \sqrt{\frac{2}{T}}N_{1s}$, $A_k = \pm\frac{\sqrt{2T}}{2}A$ and $\theta_k = \theta$, we get

$$R_{1k}^{\ 2} = (2A_k \cos\theta_k + n_{1c_k})^2 + (-2A_k \sin\theta_k + n_{1s_k})^2. \tag{26}$$

For a Rician fading channel A_k is a Rician random variable with parameters α and σ. $A_k \cos\theta_k$ is a Gaussian random variable with mean $\alpha_c = \alpha\cos\phi$ where ϕ is an arbitrary phase and variance σ^2, $A_k \sin\theta_k$ is a Gaussian random variable with mean $\alpha_s = -\alpha\sin\phi$ and variance σ^2, $2A_k\cos\theta_k$ is a Gaussian random variable with mean $\alpha_c' = 2\alpha\cos\phi$ and variance $\sigma'^2 = 4\sigma^2$ and $2A_k \sin\theta_k$ is a Gaussian random variable with mean $\alpha_s' = -2\alpha\sin\phi$ and variance $\sigma'^2 = 4\sigma^2$ [4]. All of these L independent random variables $2A_k\cos\theta_k$ for k=1, 2,..., L have a mean of α_c' and all of the L independent random variables $2A_k\sin\theta_k$ for k=1, 2,..., L have a mean of α_s'. The zero mean Gaussian random variables n_{1c_k} and n_{1s_k} for k=1, 2,..., L are independent with variance

$\sigma_n^{\ 2} = N_0$. Therefore the probability density function of V_1 is non-central Chi-square distribution with 2L degrees of freedom [3],

$$f_{V_1}(v_1) = \frac{1}{2\sigma_1^{\ 2}} \left(\frac{v_1}{s^2} \right)^{\frac{L-1}{2}} \exp\left(-\frac{(v_1 + s^2)}{2\sigma_1^{\ 2}} \right) I_{L-1}\left(\frac{\sqrt{v_1}\, s}{\sigma_1^{\ 2}} \right) \quad , v_1 \geq 0 \ , \tag{27}$$

where

$$\sigma_1^{\ 2} = \sigma'^2 + \sigma_n^{\ 2} = 4\sigma^2 + \sigma_n^{\ 2} \tag{28}$$

and

$$s^2 = \sum_{i=1}^{L} \alpha_c'^{\ 2} + \sum_{i=1}^{L} \alpha_s'^{\ 2} = L(2\alpha\cos\phi)^2 + L(-2\alpha\sin\phi)^2 = 4L\alpha^2. \tag{29}$$

We denote the average energy per diversity channel as

$$\overline{E} = \alpha^2 + 2\sigma^2. \tag{30}$$

The bit energy is related to the energy per diversity channel by

$$\overline{E_b} = L\overline{E} \quad \Rightarrow \quad \overline{E} = \frac{\overline{E_b}}{L} \ . \tag{31}$$

For the non-signal branch, the decision variable V_2 is sum of the squares of 2L Gaussian random variables with zero mean and variance $\sigma_n^{\ 2}$. Hence, the non-signal branch output is given by

$$V_2 = \sum_{k=1}^{L} R_{2k}^{\ 2} = \sum_{k=1}^{L} \left(n_{2c_k}^{\ 2} + n_{2s_k}^{\ 2} \right), \tag{32}$$

where

$$R_{2k}^{\ 2} = n_{2ck}^{\ 2} + n_{2s_k}^{\ 2} \tag{33}$$

and

$$\sigma_2{}^2 = \mathrm{var}(n_{2c_k}{}^2) = \mathrm{var}(n_{2s_k}{}^2) = \sigma_n{}^2 = N_0 . \tag{34}$$

Hence the probability density function of V_2 is Chi-Square with 2L degrees of freedom which is given by [3]

$$f_{V_2}(v_2) = \frac{1}{\left(2\sigma_2{}^2\right)^L (L-1)!} v_2{}^{L-1} \exp\left(-\frac{v_2}{2\sigma_2{}^2}\right) \quad , \; v_2 \geq 0 \; . \tag{35}$$

B. BIT ERROR PROBABILITY

We now have the decision variables for both the signal and the non-signal branches under the assumption that $b_i = 0$ is transmitted. The conditional probability of correct detection is defined by [2]

$$\Pr\{Correct\ Detection \mid V_1 = v_1\} = \Pr(V_2 < V_1 | V_1 = v_1). \tag{36}$$

Conversely, when $b_i = 1$ is transmitted, due to the symmetry of the receiver branches, the conditional probability of correct detection is described as

$$\Pr\{Correct\ Detection \mid V_2 = v_2\} = \Pr(V_1 < V_2 | V_2 = v_2). \tag{37}$$

Writing out (36) we obtain

$$\Pr\{V_2 < V_1 \mid V_1 = v_1\} = \int_0^{V_1} f_{V_2}(v_2)\, dv_2$$

$$= \int_0^{V_1} \frac{1}{(2\sigma_2{}^2)^L (L-1)!} v_2{}^{L-1} \exp\left(-\frac{v_2}{2\sigma_2{}^2}\right) dv_2 . \tag{38}$$

By using the identity [4]

$$\int_0^u x^n \exp(-\mu x)\,dx = \frac{n!}{\mu^{n+1}} - \exp(-\mu u)\sum_{k=0}^{n} \frac{n!}{k!}\,\frac{u^k}{\mu^{n-k+1}} \quad , \quad u,\mu > 0 \tag{39}$$

and setting $u = v_1$, $n = L-1$ and $\mu = \dfrac{1}{2\sigma_2^{\,2}}$, we get

$$\Pr\{V_2 < V_1 \mid V_1 = v_1\} = \frac{1}{(2\sigma_2^{\,2})^L (L-1)!} \times \left[\frac{(L-1)!}{\left(\frac{1}{2\sigma_2^{\,2}}\right)^L} - \exp\left(-\frac{v_1}{2\sigma_2^{\,2}}\right) \sum_{k=0}^{L-1} \frac{(L-1)!}{k!} \frac{v_1^{\,k}}{\left(\frac{1}{2\sigma_2^{\,2}}\right)^{L-k}} \right]$$

$$= 1 - \exp\left(-\frac{v_1}{2\sigma_2^{\,2}}\right) \sum_{k=0}^{L-1} \frac{1}{k!} \left(\frac{v_1}{2\sigma_2^{\,2}}\right)^k . \tag{40}$$

This result is inserted into the probability of correct detection expression given by

$$P_c = \Pr\{\textit{Correct Detection}\} = \int_0^\infty \Pr\{\, V_2 < V_1 \mid V_1 = v_1 \,\}\, f_{V_1}(v_1)\; dv_1 \tag{41}$$

$$= \int_0^\infty \left[1 - \exp\left(-\frac{v_1}{2\sigma_2^{\,2}}\right) \sum_{k-0}^{L-1} \frac{1}{k!} \left(\frac{v_1}{2\sigma_2^{\,2}}\right)^k \right] \frac{1}{2\sigma_1^{\,2}} \left(\frac{v_1}{s^2}\right)^{\frac{L-1}{2}} \times \exp\left(-\frac{(v_1 + s^2)}{2\sigma_1^{\,2}}\right) I_{L-1}\left(\frac{\sqrt{v_1}\, s}{\sigma_1^{\,2}}\right) dv_1 \; . \tag{42}$$

Expanding this expression leads to

$$P_c = \int_0^\infty \frac{1}{2\sigma_1^{\,2}} \left(\frac{v_1}{s^2}\right)^{\frac{L-1}{2}} \exp\left(-\frac{(v_1+s^2)}{2\sigma_1^{\,2}}\right) I_{L-1}\left(\frac{\sqrt{v_1}\, s}{\sigma_1^{\,2}}\right) dv_1 - \int_0^\infty \exp\left(-\frac{v_1}{2\sigma_2^{\,2}}\right) \sum_{k=0}^{L-1} \frac{1}{k!} \left(\frac{v_1}{2\sigma_2^{\,2}}\right)^k \frac{1}{2\sigma_1^{\,2}} \left(\frac{v_1}{s^2}\right)^{\frac{L-1}{2}} \exp\left(-\frac{(v_1+s^2)}{2\sigma_1^{\,2}}\right) I_{L-1}\left(\frac{\sqrt{v_1}\, s}{\sigma_1^{\,2}}\right) dv_1 \, . \tag{43}$$

In order to simplify, we use the identities [5]

i) $I_n(z) = (-j)^n J_n(jz)$, (44)

ii) $$\int_0^\infty x^{m+\frac{n}{2}} \exp(-\alpha x) J_n\left(2\beta\sqrt{x}\right) dx = \frac{m!\,\beta^n}{\alpha^{m+n+1}} \exp\left(-\frac{\beta^2}{\alpha}\right) L_m^n\left(\frac{\beta^2}{\alpha}\right), \quad (45)$$

iii) $$L_m^n\left(\frac{\beta^2}{\alpha}\right) = \sum_{p=0}^{m} \frac{(-1)^p}{p!} \binom{m+n}{m-p} \left(\frac{\beta^2}{\alpha}\right)^p. \quad (46)$$

The first integral expression in (43) is the integral of a probability density function given by

$$\int_0^\infty \frac{1}{2\sigma_1^2}\left(\frac{v_1}{s^2}\right)^{\frac{L-1}{2}} \exp\left(-\frac{(v_1+s^2)}{2\sigma_1^2}\right) I_{L-1}\left(\frac{\sqrt{v_1}\,s}{\sigma_1^2}\right) dv_1 = 1. \quad (47)$$

The second integral becomes

$$\begin{aligned}
&\int_0^\infty \exp\left(-\frac{v_1}{2\sigma_2^2}\right) \sum_{k=0}^{L-1} \frac{1}{k!}\left(\frac{v_1}{2\sigma_2^2}\right)^k \frac{1}{2\sigma_1^2}\left(\frac{v_1}{s^2}\right)^{\frac{L-1}{2}} \exp\left(-\frac{(v_1+s^2)}{2\sigma_1^2}\right) I_{L-1}\left(\frac{\sqrt{v_1}\,s}{\sigma_1^2}\right) dv_1 = \\
&\frac{1}{\left(1+\dfrac{\sigma_1^2}{\sigma_2^2}\right)^L} \exp\left(-\frac{s^2}{2\sigma_1^2}\left(1-\frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\right)\right) \sum_{k=0}^{L-1} \left(1+\frac{\sigma_2^2}{\sigma_1^2}\right)^{-k} \\
&\times \sum_{m=0}^{k} \frac{(-1)^m}{m!} \binom{k+L-1}{k-m} \left[\frac{-s^2}{2\sigma_1^2} \cdot \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\right]^m.
\end{aligned} \quad (48)$$

Substituting (47) and (48) into (43), the probability of correct detection is obtained as

$$P_c = 1 - \frac{1}{\left(1+\frac{\sigma_1^{\,2}}{\sigma_2^{\,2}}\right)^L} \exp\left(-\frac{s^2}{2\sigma_1^{\,2}}\left(1-\frac{\sigma_2^{\,2}}{\sigma_1^{\,2}+\sigma_2^{\,2}}\right)\right)\sum_{k=0}^{L-1}\left(1+\frac{\sigma_2^{\,2}}{\sigma_1^{\,2}}\right)^{-k}$$

$$\times\sum_{m=0}^{k}\frac{1}{m!}\binom{k+L-1}{k-m}\left[\frac{s^2}{2\sigma_1^{\,2}}\cdot\frac{\sigma_2^{\,2}}{\sigma_1^{\,2}+\sigma_2^{\,2}}\right]^m . \tag{49}$$

The probability of error expression is obtained from $P_e = 1 - P_c$. So

$$P_e = 1 - P_c = \frac{1}{\left(1+\frac{\sigma_1^{\,2}}{\sigma_2^{\,2}}\right)^L} \exp\left(-\frac{s^2}{2\sigma_1^{\,2}}\left(1-\frac{\sigma_2^{\,2}}{\sigma_1^{\,2}+\sigma_2^{\,2}}\right)\right)\sum_{k=0}^{L-1}\left(1+\frac{\sigma_2^{\,2}}{\sigma_1^{\,2}}\right)^{-k}$$

$$\times\sum_{m=0}^{k}\frac{1}{m!}\binom{k+L-1}{k-m}\left[\frac{s^2}{2\sigma_1^{\,2}}\cdot\frac{\sigma_2^{\,2}}{\sigma_1^{\,2}+\sigma_2^{\,2}}\right]^m . \tag{50}$$

From (28), (29) and (34), we can express $\sigma_1^{\,2}$, s^2 and $\sigma_2^{\,2}$ as follows; $\sigma_1^{\,2} = 4\sigma^2 + N_0$, $s^2 = 4L\alpha^2$ and $\sigma_2^{\,2} = N_0$, respectively. Then using the average energy per diversity channel $\overline{E}$ in (30), we obtain

$$\overline{E} = \alpha^2 + 2\sigma^2 \quad \Rightarrow \quad 2\sigma^2 = \frac{\overline{E}}{\gamma+1}, \tag{51}$$

where $\gamma = \alpha^2 / 2\sigma^2$ is the direct-to-diffuse ratio given in (3). The probability of error in terms of the average energy per diversity channel is given by

$$P_e = \exp\left(-L\frac{\overline{E}}{N_0}\frac{\gamma}{\gamma+1+\frac{\overline{E}}{N_0}} \right) \sum_{k=0}^{L-1} \frac{\left(1+2\frac{\overline{E}}{N_0}\cdot\frac{1}{\gamma+1}\right)^k}{\left(2+2\frac{\overline{E}}{N_0}\cdot\frac{1}{\gamma+1}\right)^{L+k}}$$
$$\times \sum_{m=0}^{k} \frac{1}{m!}\binom{L-1+k}{k-m}\left[L\frac{\overline{E}}{N_0}\cdot\frac{\gamma(\gamma+1)}{\left(\gamma+1+\frac{\overline{E}}{N_0}\right)\left(\gamma+1+2\frac{\overline{E}}{N_0}\right)} \right]^m . \quad (52)$$

By using (31), the probability of bit error for a noncoherent DPSK receiver with equal gain combining in a Rician fading channel in terms of the bit energy-to-noise density ratio $\overline{E_b} / N_0$ is given by

$$P_b = \exp\left(-\frac{\overline{E_b}}{N_0}\frac{\gamma}{\gamma+1+\frac{1}{L}\frac{\overline{E_b}}{N_0}} \right) \sum_{k=0}^{L-1} \frac{\left(1+\frac{2}{L}\frac{\overline{E_b}}{N_0}\frac{1}{\gamma+1}\right)^k}{\left(2+\frac{2}{L}\frac{\overline{E_b}}{N_0}\frac{1}{\gamma+1}\right)^{L+k}}$$
$$\times \sum_{m=0}^{k} \frac{1}{m!}\binom{L-1+k}{k-m}\left[\frac{\overline{E_b}}{N_0}\frac{\gamma(\gamma+1)}{\left(\gamma+1+\frac{1}{L}\frac{\overline{E_b}}{N_0}\right)\left(\gamma+1+\frac{2}{L}\frac{\overline{E_b}}{N_0}\right)} \right]^m . \quad (53)$$

To verify this result, we can set L=1 and $s^2 = 4L\alpha^2 = 0$ (i.e.; there is no direct signal component) and we obtain the well-known expression of the probability of bit error for DPSK in Rayleigh fading channel which is given by [3]

$$P_b = \frac{1}{2+\frac{2\overline{E_b}}{N_0}} . \quad (54)$$

Similarly, if we set L=1, we obtain the well known expression for probability of bit error of DPSK in Rician fading channel given by [6]

$$P_b = \frac{1}{2}\frac{\gamma+1}{\gamma+1+\frac{\overline{E_b}}{N_o}} \exp\left(-\gamma\left(1-\frac{\gamma+1}{\gamma+1+\frac{\overline{E_b}}{N_o}}\right)\right). \tag{55}$$

The numerical results of this analysis are presented in chapter V. In the next chapter, the SC technique is analyzed and the bit error probability expression is obtained.

III. SELECTION COMBINING

Selection Combining (SC) is a predetection combining technique that takes the signal that has the largest amplitude (hence largest signal-to-noise ratio) in the L branches at the output of the combiner preceeding the DPSK demodulator. We define

$$V = \max\left(Y_1, Y_2, Y_3, \ldots, Y_L\right), \tag{56}$$

where Y_k, $(k = 1,2,\ldots,L)$, is the squared amplitude of the signal received by the k^{th} channel. The receiver for SC is shown in Fig. 3. The derivation of the probability density function for the largest random variable V is given in the Appendix.

A. PROBABILITY DENSITY FUNCTION OF THE DECISION VARIABLES

The pdf of Y_k is non-central Chi-square and is given by [3]

$$f_{Y_k}(y_k) = \frac{1}{2\sigma^2}\exp\left(-\frac{\left(y_k + s^2\right)}{2\sigma^2}\right) I_0\left(\frac{\sqrt{y_k}\, s}{\sigma^2}\right) \; ; \; y_k \geq 0 \quad . \tag{57}$$

The cumulative distribution function (cdf) of this pdf is [3]

$$F_{Y_k}(y_k) = 1 - Q\left(\frac{s}{\sigma^2}, \frac{\sqrt{y_k}}{\sigma^2}\right). \tag{58}$$

Q(a,b) in (58) is known as Marcum's Q function and defined as [3]

$$Q(a,b) = \int_b^\infty x \exp\left(-\frac{\left(x^2 + a^2\right)}{2}\right) I_0(ax)\,dx \tag{59}$$

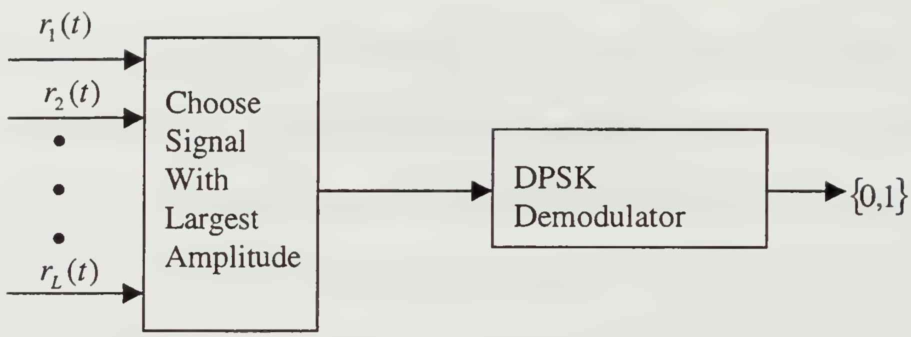

Figure 3. Selection Combining with L^{th} order diversity

and

$$Q(a,0)=1 \qquad Q(0,b)=\exp\left(-\frac{b^2}{2}\right). \tag{60}$$

Now we can define the probability density function for the largest diversity branch by using (A.14) as derived in the Appendix [7]

$$f_V(v)=L\, f_{Y_k}(v)F_{Y_k}{}^{L-1}(v). \tag{61}$$

Substituting (57) and (58) into (61), we get

$$f_V(v)=\frac{L}{2\sigma^2}\exp\left(-\frac{(v+s^2)}{2\sigma^2}\right)I_0\left(\frac{\sqrt{v}\,s}{\sigma^2}\right)\left[1-Q\left(\frac{s}{\sigma},\frac{\sqrt{v}}{\sigma}\right)\right]^{L-1}. \tag{62}$$

B. BIT ERROR PROBABILITY

In deriving the bit-error performance, the bit error rate expression of DPSK for a non-fading channel is used which is given by [3]

$$P_b(v) = \frac{1}{2}\exp\left(-\frac{v}{N_0}\right), \tag{63}$$

where v / N_0 is the signal-to-noise ratio per channel at the input of the DPSK demodulator. To obtain the error probability expression, we evaluate an integral of the form

$$P_b = \int_0^\infty P_b(v)\, f_V(v)\; dv\,. \tag{64}$$

Substituting (62) and (63) into (64), the probability of bit error expression for SC leads to

$$\begin{aligned}
P_b &= \int_0^\infty \frac{1}{2}\exp\left(-\frac{v}{N_0}\right)\frac{L}{2\sigma^2}\exp\left(-\frac{\left(v+s^2\right)}{2\sigma^2}\right) \\
&\quad \times I_0\left(\frac{\sqrt{v}\, s}{\sigma^2}\right)\left[1-Q\left(\frac{s}{\sigma},\frac{\sqrt{v}}{\sigma}\right)\right]^{L-1} dv \\
&= \frac{L}{4\sigma^2}\exp\left(-\frac{s^2}{2\sigma^2}\right)\int_0^\infty \exp\left(-\frac{v}{N_0}\left(1+\frac{N_0}{2\sigma^2}\right)\right) \\
&\quad \times I_0\left(\frac{\sqrt{v}\, s}{\sigma^2}\right)\left[1-Q\left(\frac{s}{\sigma},\frac{\sqrt{v}}{\sigma}\right)\right]^{L-1} dv.
\end{aligned} \tag{65}$$

Changing the variable, $u = \dfrac{v}{N_0}$ in (65), we get

$$\begin{aligned}
P_b &= \frac{N_0}{4\sigma^2}\exp\left(-\frac{s^2}{2\sigma^2}\right)\int_0^\infty \exp\left(-u\left(1+\frac{N_0}{2\sigma^2}\right)\right) \\
&\quad \times I_0\left(\frac{\sqrt{uN_0}\, s}{\sigma^2}\right)\left[1-Q\left(\frac{s}{\sigma},\frac{\sqrt{uN_0}}{\sigma}\right)\right]^{L-1} du\,.
\end{aligned} \tag{66}$$

The direct-to-diffuse signal power ratio of the signal is defined as

$$\gamma = \frac{s^2}{2\sigma^2}. \tag{67}$$

Hence the energy per diversity channel is described by (eq. 30)

$$\overline{E} = s^2 + 2\sigma^2 = 2\sigma^2\left(\frac{s^2}{2\sigma^2} + 1\right) = 2\sigma^2(\gamma + 1) \quad \Rightarrow \quad \sigma^2 = \frac{\overline{E}}{2(\gamma + 1)}. \tag{68}$$

Rewriting the parameters σ^2 and s^2 in (66) by using (67) and (68), the probability of the bit error expression in terms of the energy per diversity channel is obtained as

$$\begin{aligned} P_b = & \frac{L}{\frac{2}{\gamma+1}\frac{\overline{E}}{N_0}} \exp(-\gamma) \int_0^\infty \exp\left(-u\left(1 + \frac{\gamma+1}{\frac{\overline{E}}{N_0}}\right)\right) I_0\left(\sqrt{\frac{4u\gamma(\gamma+1)}{\frac{\overline{E}}{N_0}}}\right) \\ & \times \left[1 - Q\left(\sqrt{2\gamma}, \sqrt{\frac{2u(\gamma+1)}{\frac{\overline{E}}{N_0}}}\right)\right]^{L-1} du. \end{aligned} \tag{69}$$

As a result, the probability of bit error for noncoherent DPSK receiver with Selection Combining in a Rician fading channel is defined as

$$\begin{aligned} P_b = & \frac{L}{\frac{2}{\gamma+1}\frac{1}{L}\frac{\overline{E_b}}{N_0}} \exp(-\gamma) \int_0^\infty \exp\left(-u\left(1 + \frac{\gamma+1}{\frac{1}{L}\frac{\overline{E_b}}{N_0}}\right)\right) I_0\left(\sqrt{\frac{4u\gamma(\gamma+1)}{\frac{1}{L}\frac{\overline{E_b}}{N_0}}}\right) \\ & \times \left[1 - Q\left(\sqrt{2\gamma}, \sqrt{\frac{2u(\gamma+1)}{\frac{1}{L}\frac{\overline{E_b}}{N_0}}}\right)\right]^{L-1} du. \end{aligned} \tag{70}$$

To evaluate this expression numerical analysis is required. Some approximations for computing Marcum's Q function can be found in the literature. In this thesis, a recursive method [8], is used because of its more stable and accurate results. For the numerical evaluation of $I_0(x)$, the exact definition

$$I_0(x)=\sum_{i=0}^{\infty}\frac{\left(\frac{x}{2}\right)^{2i}}{(i!)^2} \tag{71}$$

can be used [9].

The numerical results of this analysis are presented in chapter V. In the next chapter, the PDSC technique is analyzed and the bit error probability expression is obtained.

IV. POST DETECTION SELECTION COMBINING

Post Detection Selection Combining (PDSC) invented by Professors Tri Ha and Ralph Hippenstiel [10] is a technique which chooses the largest values from the signal branch and the non-signal branch as the decision variables in a DPSK noncoherent demodulator as shown in Fig. 4. Between the L diversity branches, the maximum amplitudes are chosen independently as the decision variables in both branches. For the signal present branch, the decision variable V_1 is given by

$$V_1 = \max\left(R_{11}^2, R_{12}^2, R_{13}^2, \ldots, R_{1L}^2\right) = \max\left(Y_{11}, Y_{12}, Y_{13}, \ldots, Y_{1L}\right). \tag{72}$$

For the non-signal branch, which corresponds to the noise only condition, the decision variable V_2 is given by

$$V_2 = \max\left(R_{21}^2, R_{22}^2, R_{23}^2, \ldots, R_{2L}^2\right) = \max\left(Y_{21}, Y_{22}, Y_{23}, \ldots, Y_{2L}\right), \tag{73}$$

where Y_{1i} and Y_{2i}, $i = (1,2,\ldots,L)$, are the amplitudes received by the i^{th} channel in both branches.

A. PROBABILITY DENSITY FUNCTION OF THE DECISION VARIABLES

For the signal branch, the probability density function of the i^{th} branch output Y_{1i} before combining, is modeled as in (27) with a non-central Chi-square distribution with L set to 1 [3];

$$f_{Y_{1i}}(y_{1i}) = \frac{1}{2\sigma_1^2}\exp\left(-\frac{\left(y_{1i} + s^2\right)}{2\sigma_1^2}\right) I_0\left(\frac{\sqrt{y_{1i}}\, s}{\sigma_1^2}\right) \; ; \quad y_{1i} \geq 0 \; . \tag{74}$$

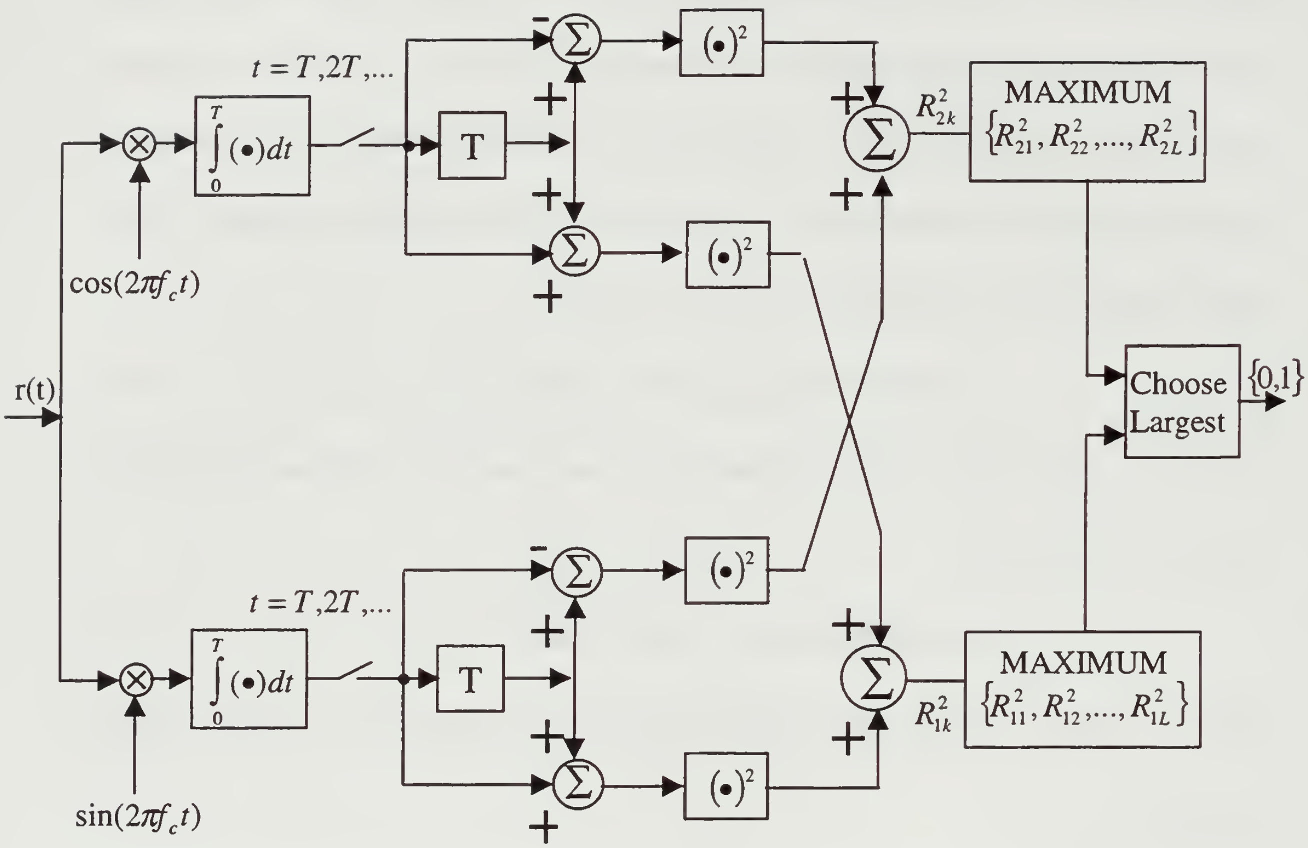

Figure 4. Noncoherent DPSK receiver for Post Detection Selection Combining (PDSC) with L^{th} order diversity

Each of the L diversity branches has the probability density function given by (74) and a corresponding cumulative distribution function, cdf, given by

$$F_{Y_{1i}}(y_{1i}) = 1 - Q\left(\frac{s}{\sigma_1}, \frac{\sqrt{y_{1i}}}{\sigma_1}\right). \tag{75}$$

Since all L branch variables are statistically independent, using (61), the pdf for the signal branch decision variable becomes

$$f_{V_1}(v_1) = L\, f_{Y_{1i}}(v_1) F_{Y_{1i}}^{\;L-1}(v_1)$$

$$= \frac{L}{2\sigma_1^{\,2}} \exp\left(-\frac{(v_1 + s^2)}{2\sigma_1^{\,2}}\right) I_0\left(\frac{\sqrt{v_1}\, s}{\sigma_1^{\,2}}\right) \left[1 - Q\left(\frac{s}{\sigma_1}, \frac{\sqrt{v_1}}{\sigma_1}\right)\right]^{L-1} \quad ; \; v_1 \geq 0. \tag{76}$$

Similarly, in the non-signal branch, under the noise only condition, the maximum amplitude is chosen as the decision variable. Before diversity combining, the i^{th} branch output Y_{2i} is modeled as in (35) with a Chi-square distribution with L set to 1:

$$f_{Y_2}(y_2) = \frac{1}{2\sigma_2^{\,2}} \exp\left(-\frac{y_2}{2\sigma_2^{\,2}}\right), \tag{77}$$

with a corresponding cdf given by

$$F_{Y_{2i}}(y_{2i}) = 1 - \exp\left(-\frac{y_{2i}}{2\sigma_2^{\,2}}\right). \tag{78}$$

The probability density function for the largest amplitude in the non-signal branch becomes

$$f_{V_2}(v_2) = L\, f_{Y_{2i}}(v_2) F_{Y_{2i}}^{\;L-1}(v_2)$$

$$= \frac{L}{2\sigma_2^{\,2}} \exp\left(-\frac{v_2}{2\sigma_2^{\,2}}\right)\left[1-\exp\left(-\frac{v_2}{2\sigma_2^{\,2}}\right)\right]^{L-1}. \tag{79}$$

B. BIT ERROR PROBABILITY

We now have the decision variables for both branches. Using the conditional bit error probability expression we get

$$P_b = \Pr\{V_2 > V_1\} = \int_{V_1=0}^{\infty}\left[\int_{V_2=v_1}^{\infty} f_{V_2}(v_2)\, dv_2\right] f_{V_1}(v_1)\, dv_1, \tag{80}$$

where

$$\int_{V_2=v_1}^{\infty} f_{V_2}(v_2)\, dv_2 = \int_{v_1}^{\infty} \frac{L}{2\sigma_2^{\,2}} \exp\left(-\frac{v_2}{2\sigma_2^{\,2}}\right)\left[1-\exp\left(-\frac{v_2}{2\sigma^{\,2}}\right)\right]^{L-1} dv_2. \tag{81}$$

Applying the binomial theorem,

$$(a+b)^N = \sum_{k=0}^{N}\binom{N}{k} a^{N-k}\, b^k \tag{82}$$

to the exponential term in (81), the exponential term can be simplified as

$$\left[1-\exp\left(-\frac{v_2}{2\sigma_2^{\,2}}\right)\right]^{L-1} = \sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k\left(\exp\left(-\frac{v_2}{2\sigma_2^{\,2}}\right)\right)^k \tag{83}$$

and (81) becomes

$$\int_{v_1}^{\infty} f_{V_2}(v_2)dv_2 = \frac{L}{2\sigma_2^{\,2}} \sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^k \int_{v_1}^{\infty} \exp\left(-v_2\left(\frac{1+k}{2\sigma_2^{\,2}}\right)\right) dv_2$$

$$= L \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \frac{1}{1+k} \exp\left(-v_1 \left(\frac{1+k}{2\sigma_2^{\,2}} \right) \right). \tag{84}$$

Substituting (76) and (84) into (80), we get

$$\begin{aligned} P_b = \int_0^\infty L \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \frac{1}{(1+k)} \exp\left(-v_1 \left(\frac{1+k}{2\sigma_2^{\,2}} \right) \right) \frac{L}{2\sigma_1^{\,2}} \exp\left(-\left(\frac{v_1 + s^2}{2\sigma_1^{\,2}} \right) \right) \\ \times I_0\left(\frac{\sqrt{v_1}\, s}{\sigma_1^{\,2}} \right) \left[1 - Q\left(\frac{s}{\sigma_1}, \frac{\sqrt{v_1}}{\sigma_1} \right) \right]^{L-1} dv_1 \end{aligned}$$

$$\begin{aligned} = \frac{L^2}{2\sigma_1^{\,2}} \exp\left(-\frac{s^2}{2\sigma_1^{\,2}} \right) \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \frac{1}{1+k} \int_0^\infty \exp\left(-v_1 \left(\frac{1+k}{2\sigma_2^{\,2}} + \frac{1}{2\sigma_1^{\,2}} \right) \right) \\ \times I_0\left(\frac{\sqrt{v_1}\, s}{\sigma_1^{\,2}} \right) \left[1 - Q\left(\frac{s}{\sigma_1}, \frac{\sqrt{v_1}}{\sigma_1} \right) \right]^{L-1} dv_1\,. \end{aligned} \tag{85}$$

Rewriting the parameters $\sigma_1^{\,2}$, $\sigma_2^{\,2}$ and s^2 as

$$\sigma_1^{\,2} = 4\sigma^2 + N_0 = 2\left(2\sigma^2\right) + N_0 = 2\left(\frac{\overline{E}}{\gamma+1} \right) + N_0 = N_0 \left(\frac{2}{\gamma+1} \frac{\overline{E}}{N_0} + 1 \right), \tag{86}$$

$$\sigma_2^{\,2} = N_0 \tag{87}$$

and

$$s^2 = 4\alpha^2\,. \tag{88}$$

Then the probability of bit error expression in terms of the energy per diversity channel is described by

$$P_b = \frac{L^2}{2N_0\left(\frac{2}{\gamma+1}\frac{\overline{E}}{N_0}+1\right)} \exp\left(-\left(\frac{2\gamma}{2+\frac{\gamma+1}{\overline{E}/N_0}}\right)\right) \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \frac{1}{1+k}$$

$$\times \int_0^\infty \exp\left(-\frac{v_1}{2N_0}\left(1+k+\frac{1}{\left(\frac{2}{\gamma+1}\frac{\overline{E}}{N_0}+1\right)}\right)\right) I_0\left(\frac{\sqrt{4\frac{v_1}{N_0}\frac{\overline{E}}{N_0}\left(\frac{\gamma}{\gamma+1}\right)}}{\left(\frac{2}{\gamma+1}\frac{\overline{E}}{N_0}+1\right)}\right) \tag{89}$$

$$\times \left[1-Q\left(\sqrt{\frac{4\gamma}{2+\frac{\gamma+1}{\overline{E}/N_0}}}, \sqrt{\frac{v_1/N_0}{\frac{2}{\gamma+1}\frac{\overline{E}}{N_0}+1}}\right)\right]^{L-1} dv_1 .$$

By a change of variable, $u = \frac{v_1}{N_0}$, we get

$$P_b = \frac{L^2}{2\left(\frac{2}{\gamma+1}\frac{\overline{E}}{N_0}+1\right)} \exp\left(-\left(\frac{2\gamma}{2+\frac{\gamma+1}{\overline{E}/N_0}}\right)\right) \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \frac{1}{1+k}$$

$$\times \int_0^\infty \exp\left(-\frac{u}{2}\left(1+k+\frac{1}{\frac{2}{\gamma+1}\frac{\overline{E}}{N_0}+1}\right)\right) I_0\left(\frac{\sqrt{4u\frac{\overline{E}}{N_0}\left(\frac{\gamma}{\gamma+1}\right)}}{\left(\frac{2}{\gamma+1}\frac{\overline{E}}{N_0}+1\right)}\right) \tag{90}$$

$$\left[1-Q\left(\sqrt{\frac{4\gamma}{2+\frac{\gamma+1}{\overline{E}/N_0}}}, \sqrt{\frac{u}{\frac{2}{\gamma+1}\frac{\overline{E}}{N_0}+1}}\right)\right]^{L-1} du .$$

Substituting $\overline{E}$ by $\overline{E_b}/L$ in (90), we get

$$P_b = \frac{L^2}{2\left(\frac{2}{\gamma+1}\frac{1}{L}\frac{\overline{E_b}}{N_0}+1\right)} \exp\left(-\left(\frac{2\gamma}{2+\frac{\gamma+1}{\frac{1}{L}\frac{\overline{E_b}}{N_0}}}\right)\right) \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \frac{1}{1+k}$$

$$\times \int_0^\infty \exp\left(-\frac{u}{2}\left(1+k+\frac{1}{\left(\frac{2}{\gamma+1}\frac{1}{L}\frac{\overline{E_b}}{N_0}+1\right)}\right)\right) I_0\left(\frac{\sqrt{4u\frac{1}{L}\frac{\overline{E_b}}{N_0}\left(\frac{\gamma}{\gamma+1}\right)}}{\left(\frac{2}{\gamma+1}\frac{1}{L}\frac{\overline{E_b}}{N_0}+1\right)}\right)$$

$$\times \left[1 - Q\left(\sqrt{\frac{4\gamma}{2+\frac{\gamma+1}{\frac{1}{L}\frac{\overline{E_b}}{N_0}}}}, \sqrt{\frac{u}{\frac{2}{\gamma+1}\frac{1}{L}\frac{\overline{E_b}}{N_0}+1}}\right)\right]^{L-1} du. \qquad (91)$$

As in selection combining, this expression is evaluated by numerical analysis. The results of this analysis are presented in the next chapter.

V. NUMERICAL RESULTS

In chapter II, III and IV the bit error rate (BER) expressions for noncoherent DPSK signals operating in a frequency non-selective, slowly fading Rician channel for EGC, SC and PDSC techniques are analytically obtained. The main objective of this thesis is to illustrate the performance by allowing the comparison of the EGC, SC and PDSC techniques. The numerical evaluation of the bit error rate expressions is performed by using MATLAB 5.1 [11] and is presented in Figures 5-26. The bit energy-to-noise ratio is chosen in the range of 6-20 dB. A direct-to-diffuse signal ratio of $\gamma = 0$ denotes a Rayleigh fading channel. A γ of 1 is effectively a Rayleigh fading (weak Rician) channel while γ of 10 is effectively a Rician (i.e.; there is a strong direct signal component) channel. Hence, in numerical evaluations, values of $\gamma = 0, 1, 5, 10$ provide sufficient detail to illustrate the performance differences.

In Figs. 5-10, the EGC, the SC and the PDSC performances of the receiver are illustrated separately for diversity orders of L=2, 3, 4, 5. As expected, the L=1 case provided the same performance for all three techniques and is not plotted.

In Figs. 11-26, the performance comparison of the EGC, the SC, and the PDSC is illustrated for diversity order of L=2, 3, 4, 5 and $\gamma = 0, 1, 5, 10$.

The results in this thesis are based on the assumption that the direct-to-diffuse signal ratio γ is the same for each diversity branch (i.e.; the average power of the diversity branches is assumed to be equal).

In Figs. 5-6, the EGC performance of the receiver is shown for diversity orders of L=2,3,4,5 and the direct-to-diffuse signal ratios of $\gamma = 1,10$. For $\gamma = 1$, which means that the channel fading is effectively Rayleigh (weak Rician), as L increases the receiver performance improves. For $\gamma = 10$, when the channel fading is effectively Rician (strong direct signal component), all BER curves illustrate better performance than $\gamma = 1$ case. For low values of E_b / N_0, the system performance with smaller diversity order L is observed to be superior to those with larger diversity order L. This trend reverses as E_b / N_0 increases. This effect is due to the noncoherent combining loss. As mentioned before, noncoherent systems are not optimal because of this phenomenon. This effect is more evident with larger L values as seen in Fig. 6. We observe the performance improvement effect of increasing L after a certain bit energy-to-noise density ratio, primarily since the noise in each diversity channel is significant at low E_b / N_0 values. As E_b / N_0 increases, the noise in each diversity channel becomes less significant. In the EGC technique, as the bit energy-to noise density ratio increases (i.e.; as noise becomes less and less significant), each diversity branch contributes positively to the overall SNR and therefore a system with a larger diversity order L will perform better than a smaller L.

Figs. 7 and 9 demonstrate the receiver performance of the SC and the PDSC techniques for $\gamma = 1$, respectively. As L increases, performance improvement is observed due to the weak Rician (i.e.; Rayleigh) fading channel. Since the direct signal component is weak, most information is obtained from the diffuse signal components. When a larger diversity order L is employed, the probability that the receiver chooses a stronger signal

branch becomes larger. In this case, performance improvement as L increases is an expected result. For the $\gamma = 10$ case (strong direct signal component), L=2 provides the best performance but we observe the slight performance improvement effect by increasing L after a certain E_b / N_0 value as in the EGC performance for $\gamma = 10$ (Fig. 6). Again the noncoherent combining loss is observed in both SC and PDSC. Increasing L does not provide significant performance improvement since a strong direct signal component is already available.

In Figs. 11-26, the performance comparison of the EGC, the SC and the PDSC techniques is illustrated for L=2,3,4,5 and $\gamma = 0,1,5,10$. As clearly observed in the figures, the PDSC technique provides a BER performance better than that of SC and inferior than that of EGC. For a particular diversity order L, as γ increases all three techniques demonstrate performance improvement since the receiver performance is greatly improved by a strong direct signal component as indicated before. When L is increased, performance improvement is observed due to an increasing number of diversity branches which permits choosing a higher amplitude diversity branch. In both increasing L and increasing γ cases, the PDSC technique shows a BER performance that is better than the SC but worse than the EGC.

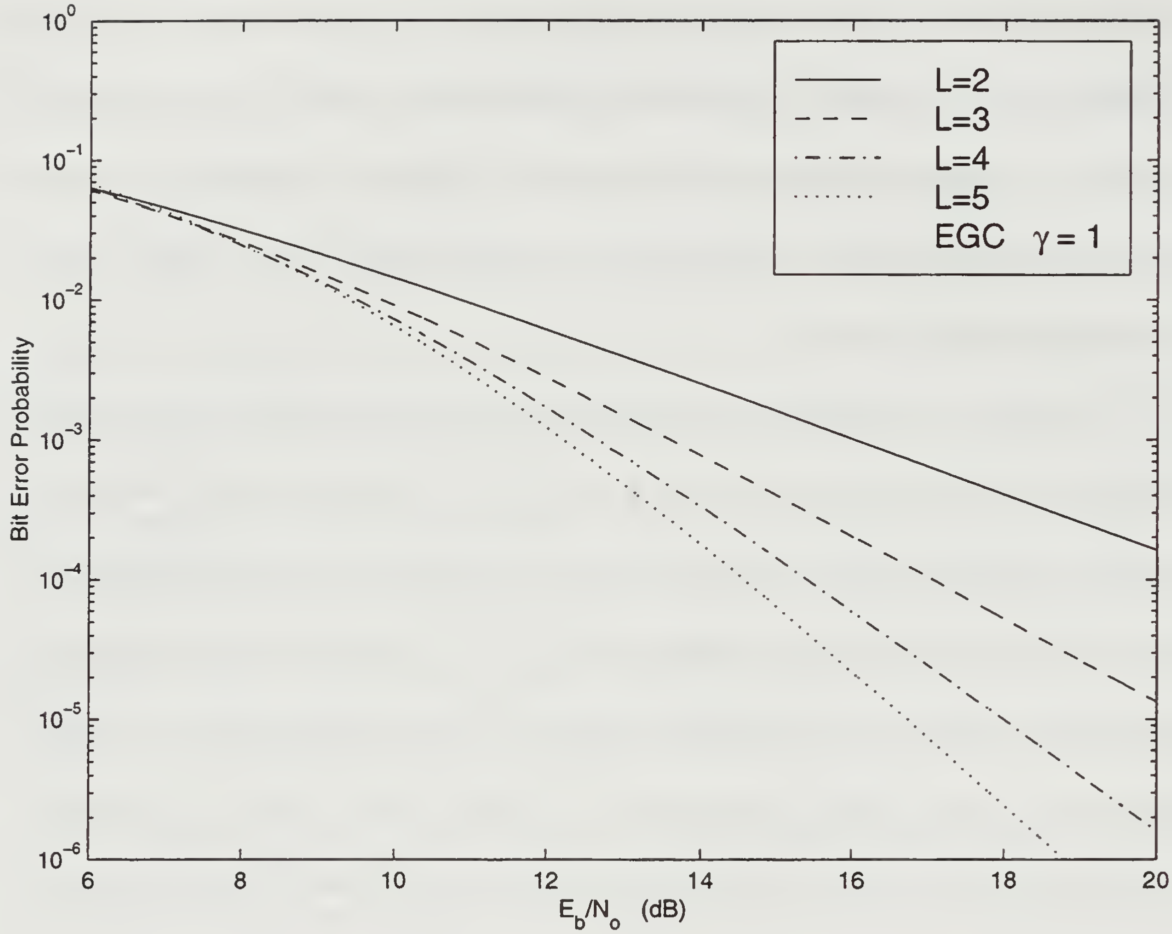

Figure 5. The performance of noncoherent DPSK in a Rician fading channel with EGC for L=2, 3, 4, 5 and $\gamma = 1$.

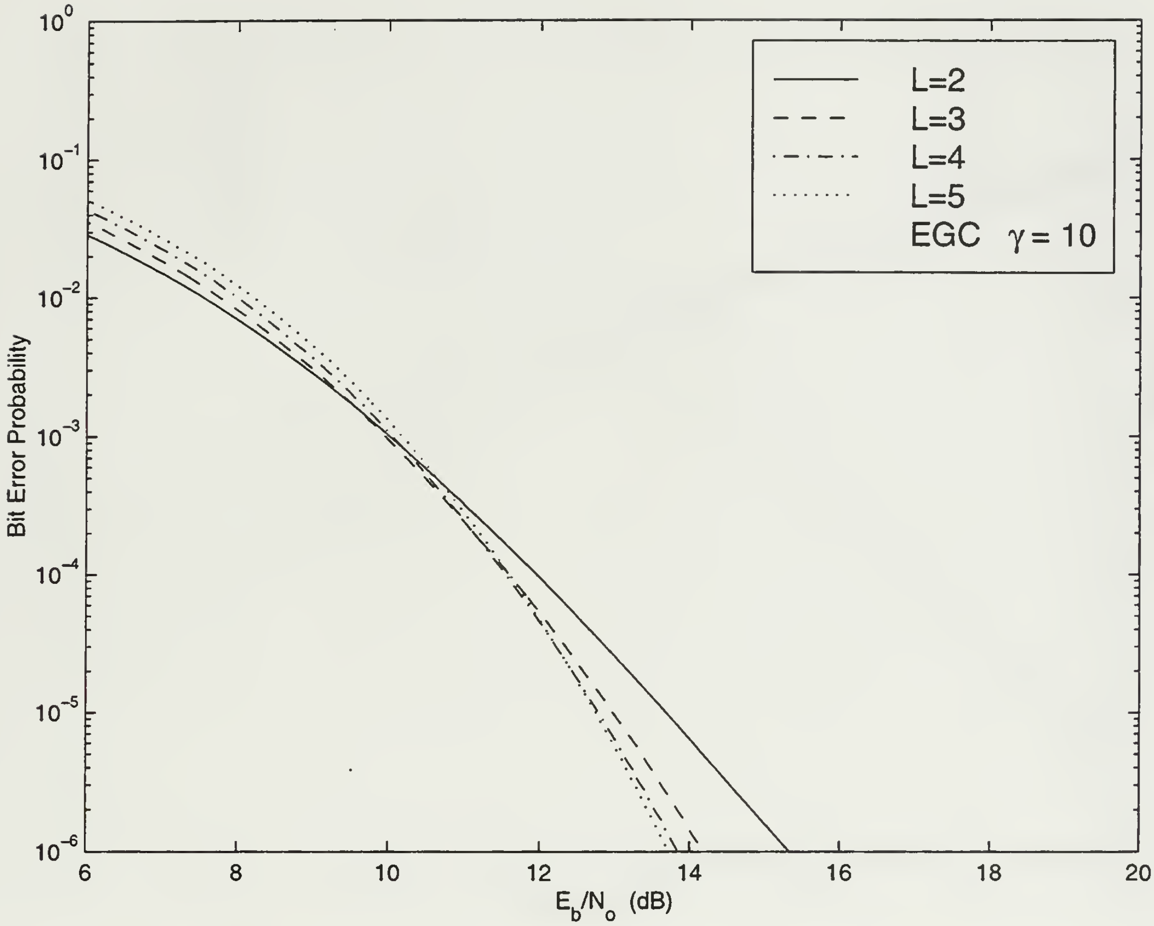

Figure 6. The performance of noncoherent DPSK in a Rician fading channel with EGC for L=2, 3, 4, 5 and $\gamma = 10$.

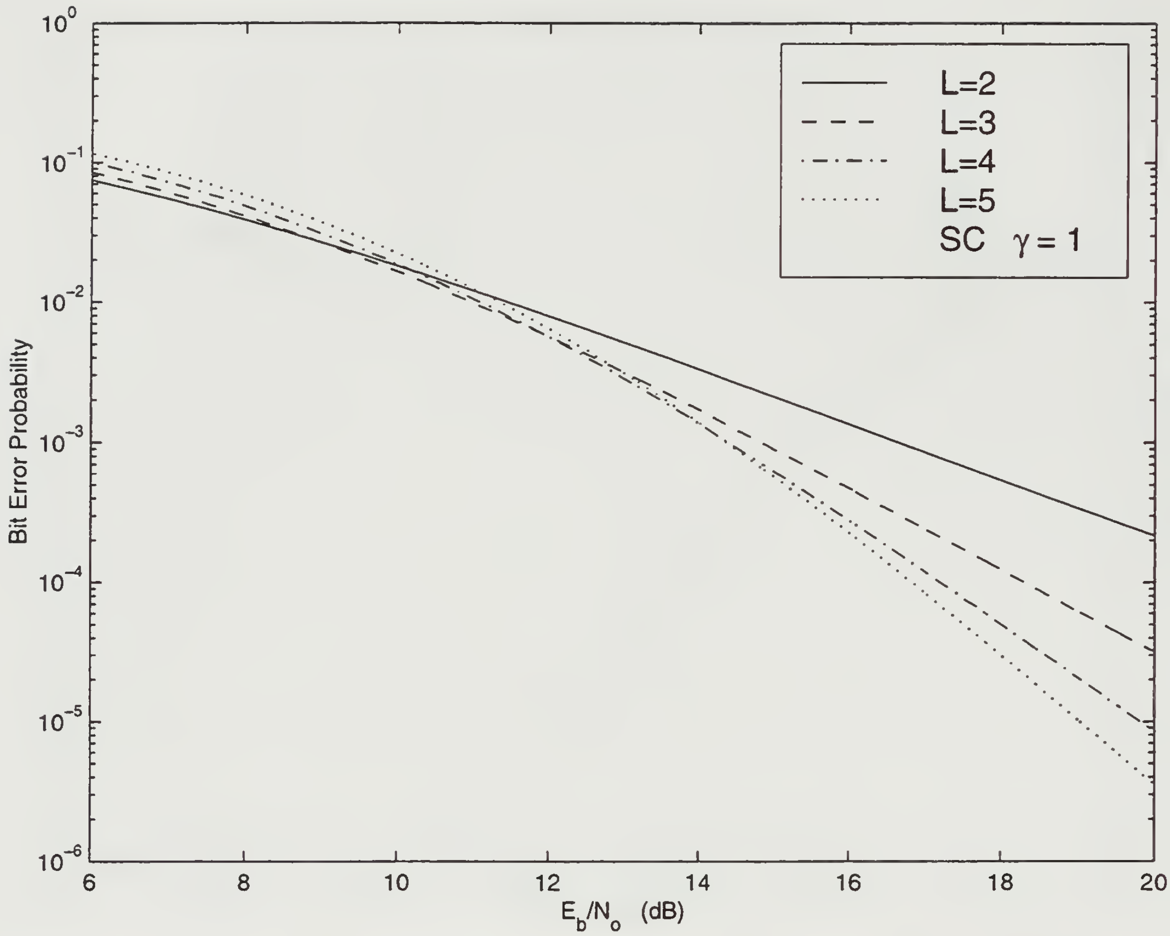

Figure 7. The performance of noncoherent DPSK in a Rician fading channel with SC for L=2, 3, 4, 5 and $\gamma = 1$.

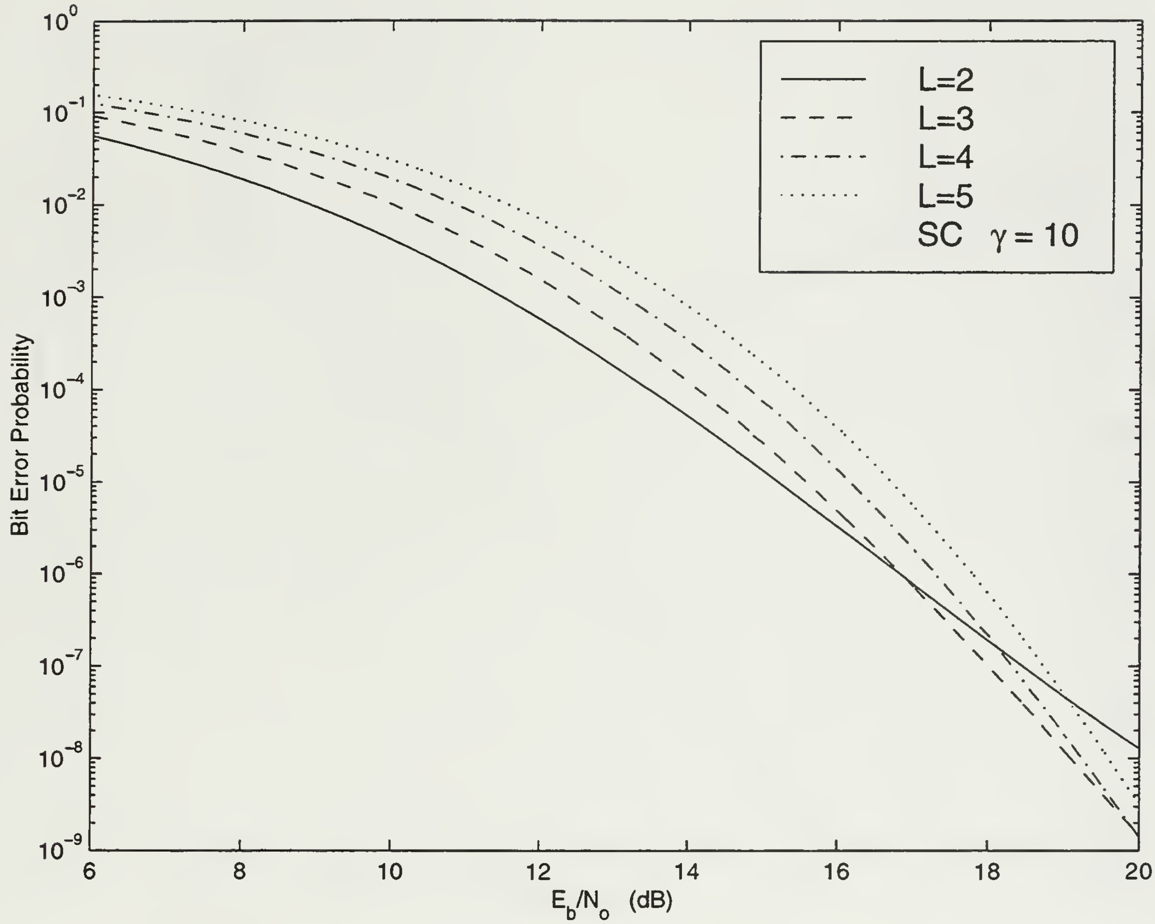

Figure 8. The performance of noncoherent DPSK in a Rician fading channel with SC for L=2, 3, 4, 5 and $\gamma = 10$.

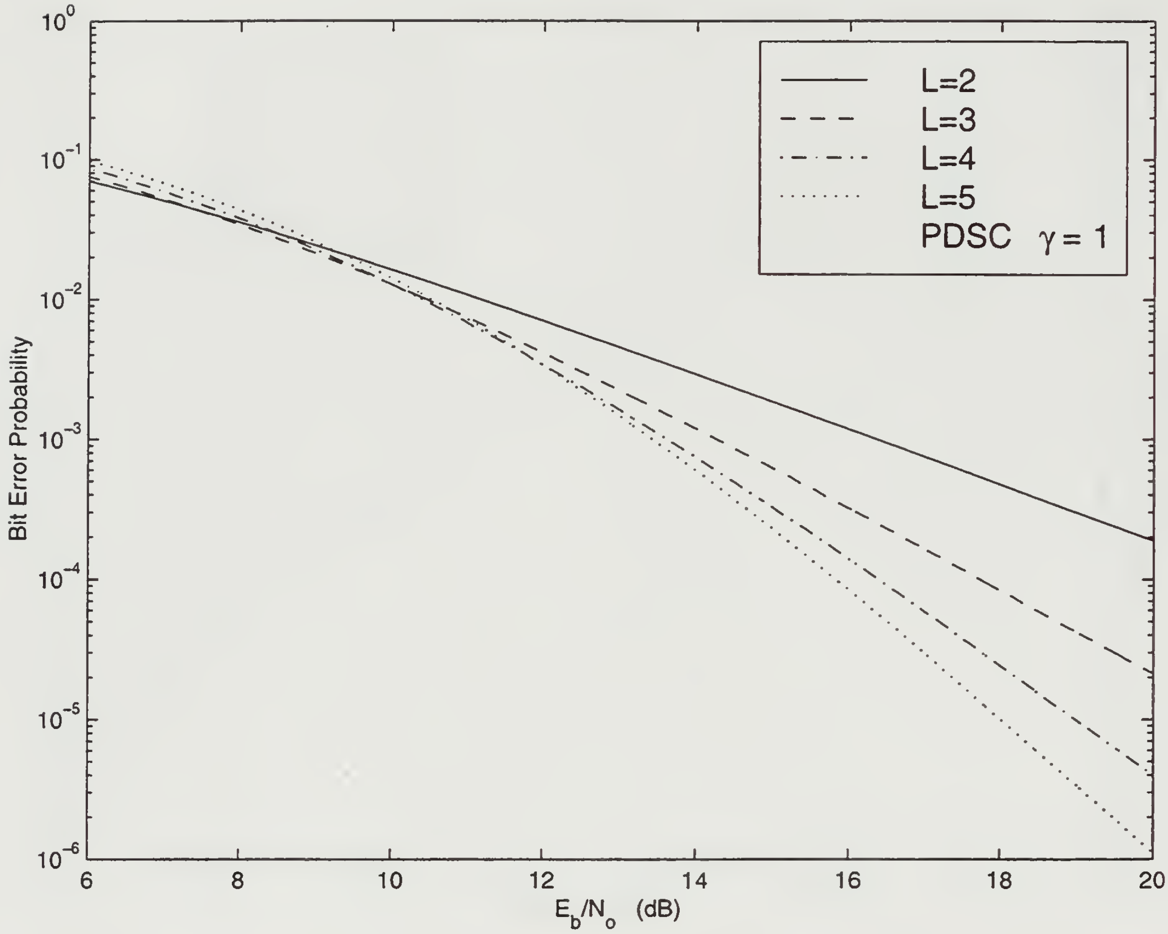

Figure 9. The performance of noncoherent DPSK in a Rician fading channel with PDSC for L=2, 3, 4, 5 and $\gamma = 1$.

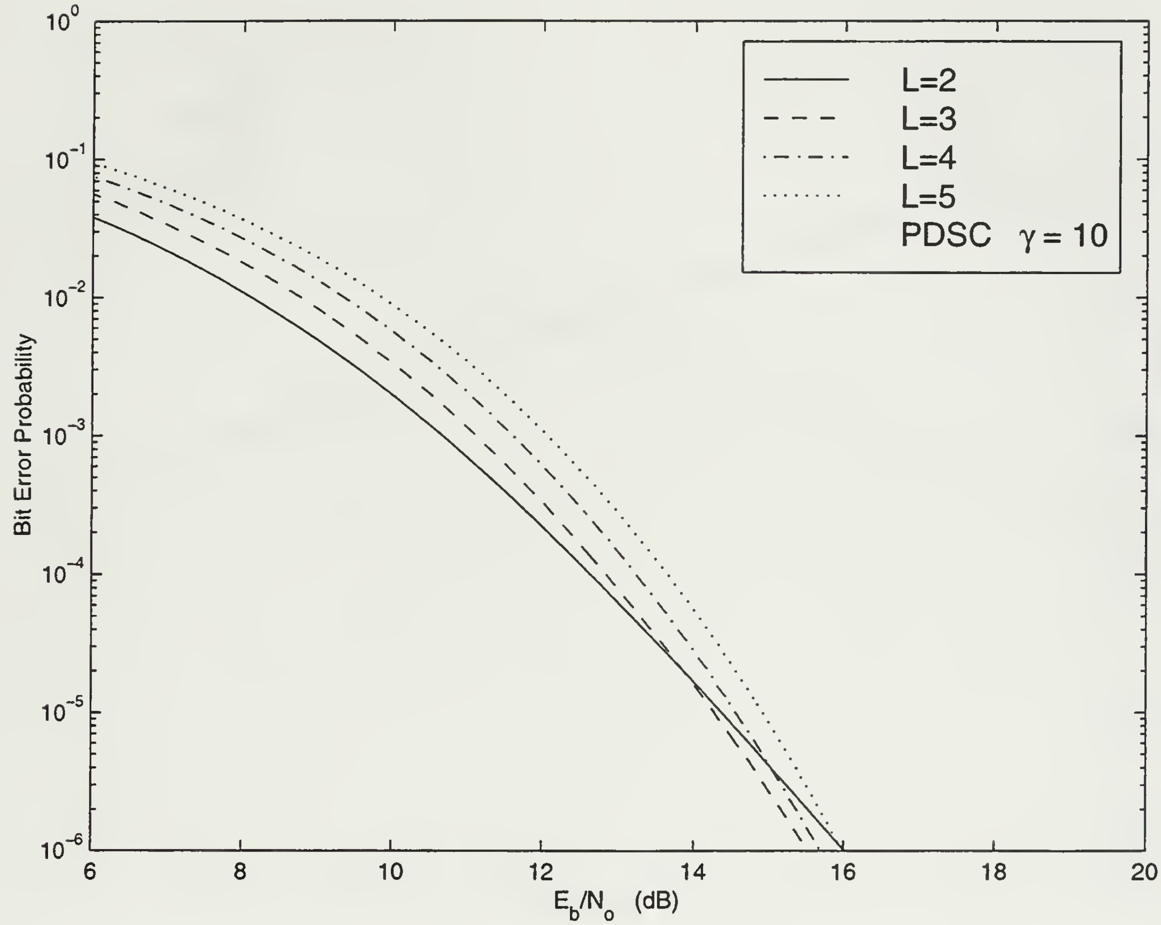

Figure 10. The performance of noncoherent DPSK in a Rician fading channel with PDSC for L=2, 3, 4, 5 and $\gamma = 10$.

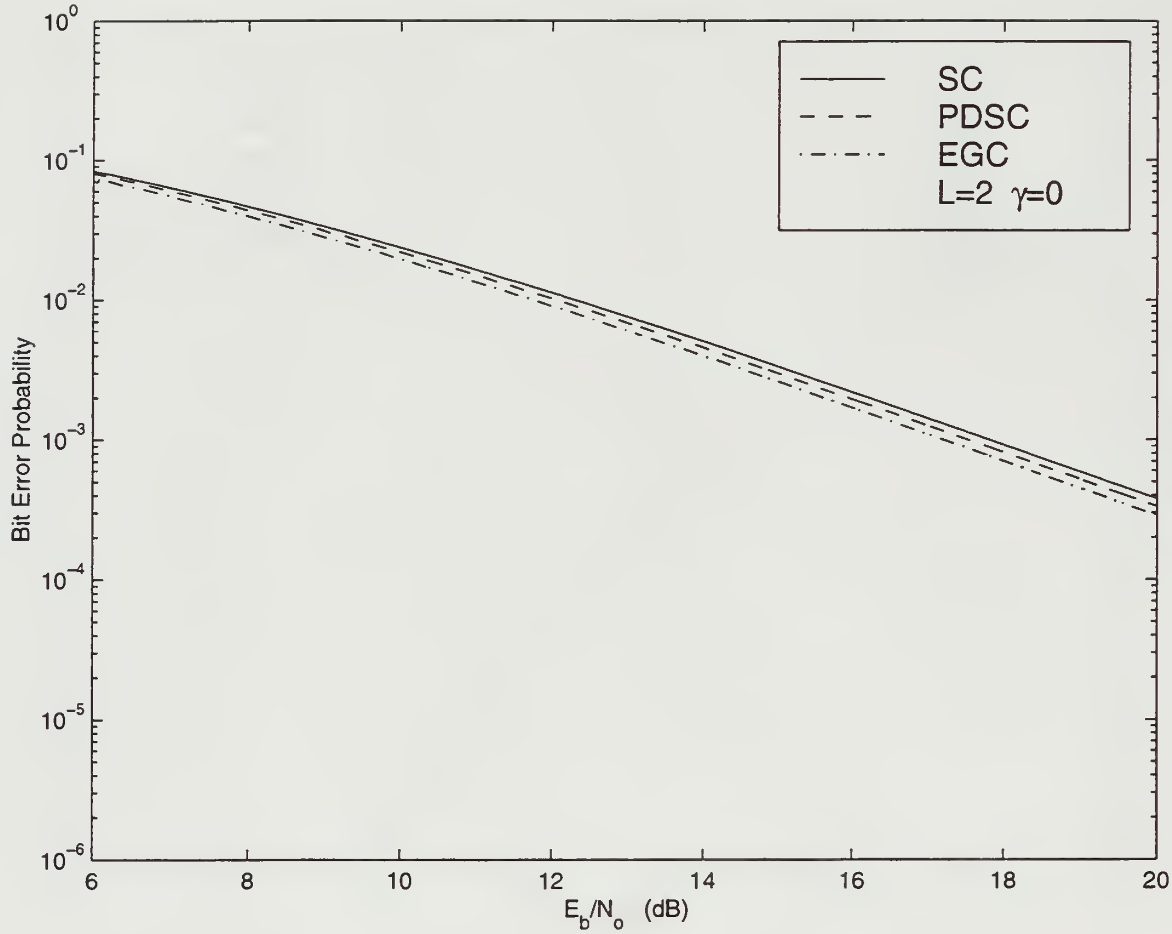

Figure 11. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=2 and $\gamma = 0$.

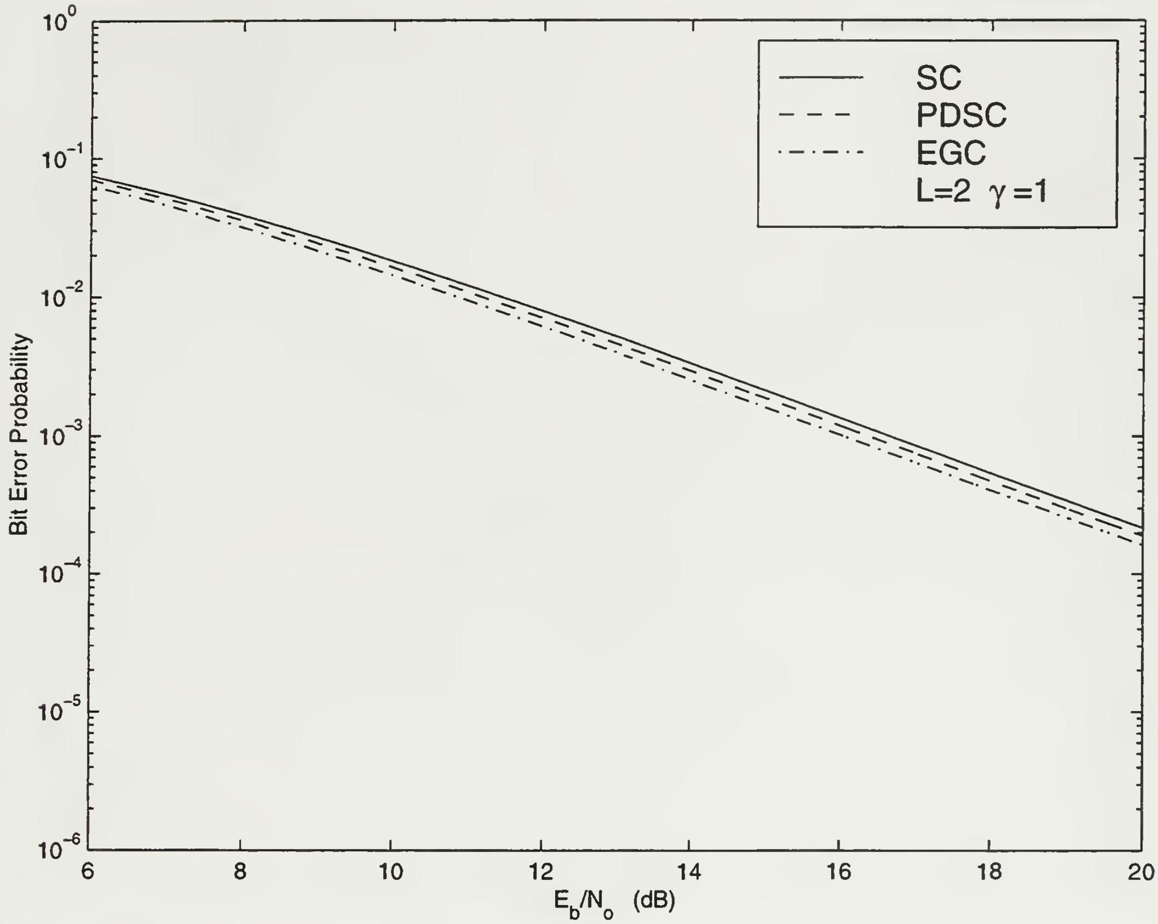

Figure 12. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=2 and $\gamma = 1$.

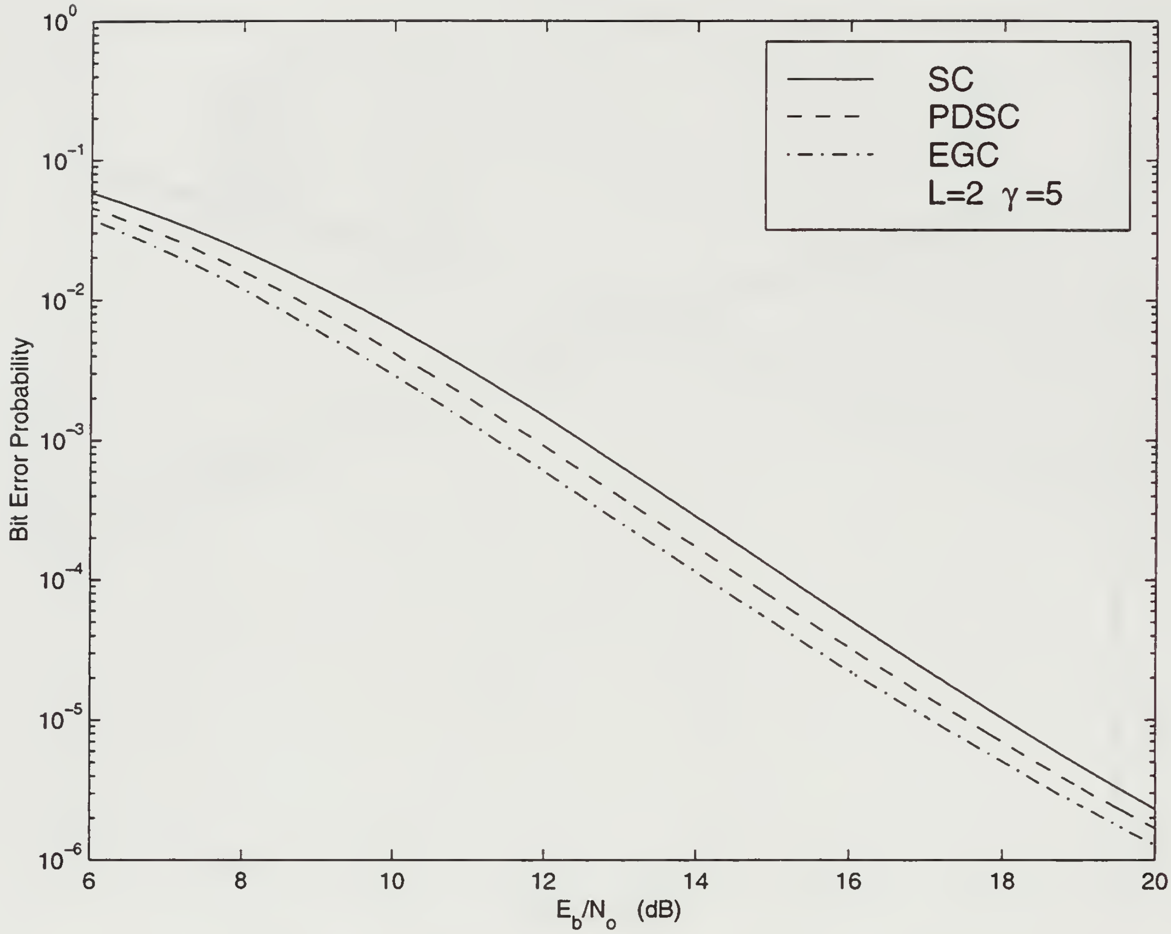

Figure 13. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=2 and $\gamma = 5$.

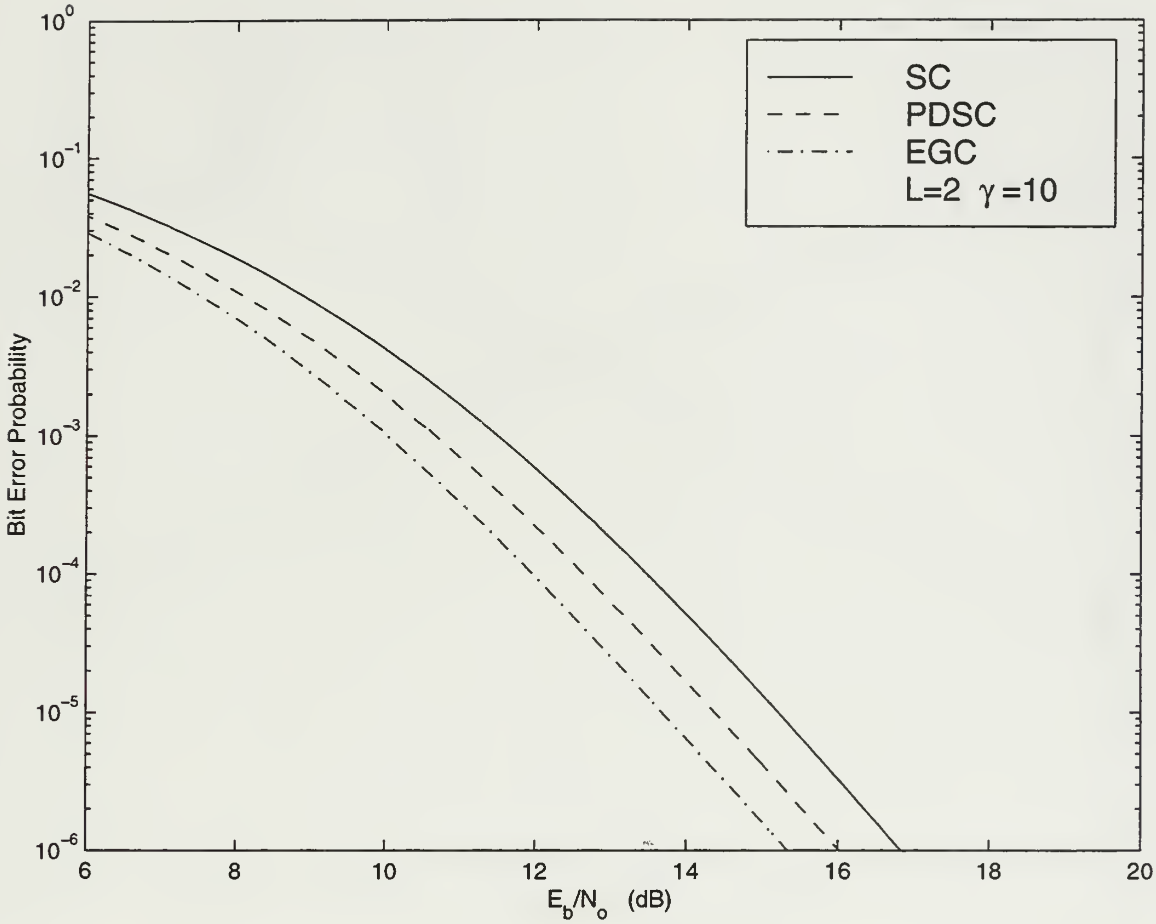

Figure 14. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=2 and $\gamma = 10$.

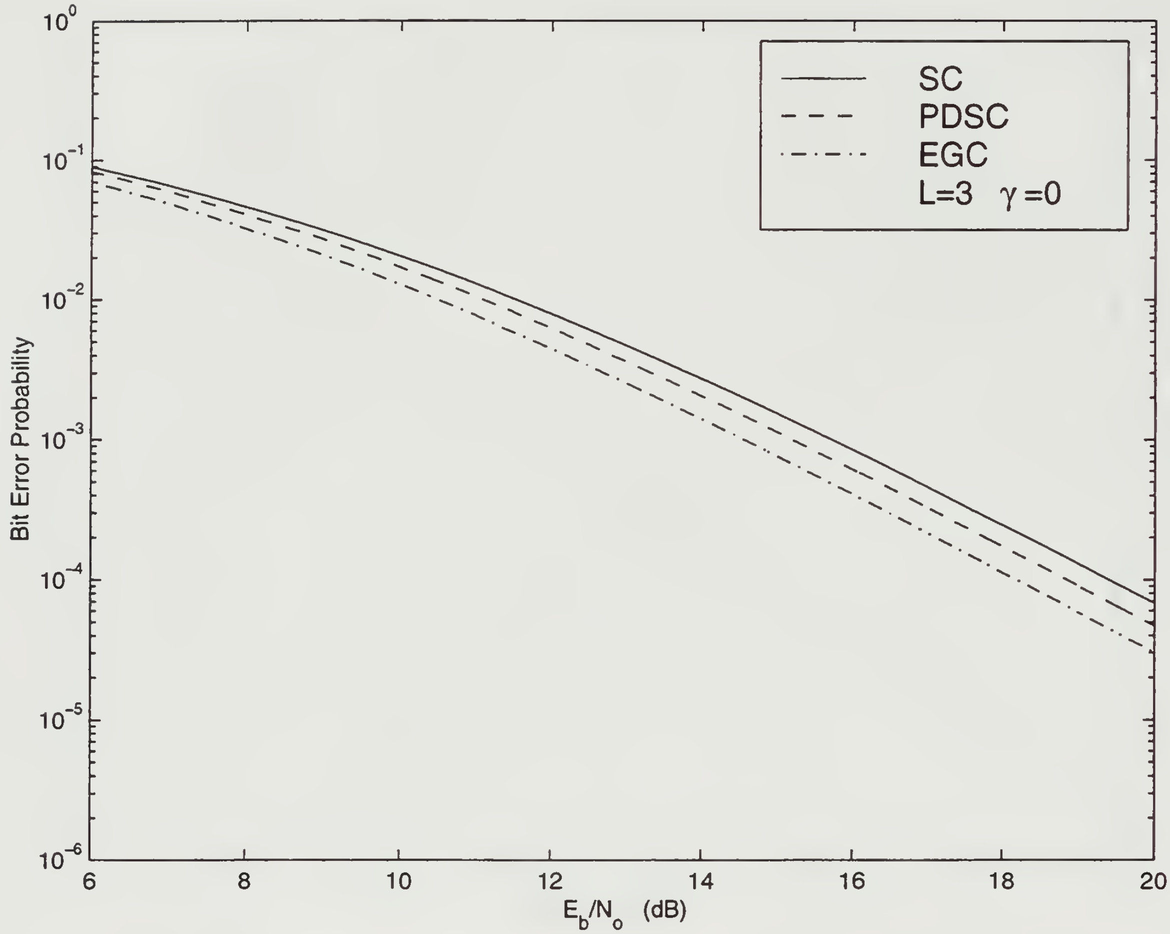

Figure 15. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=3 and $\gamma = 0$.

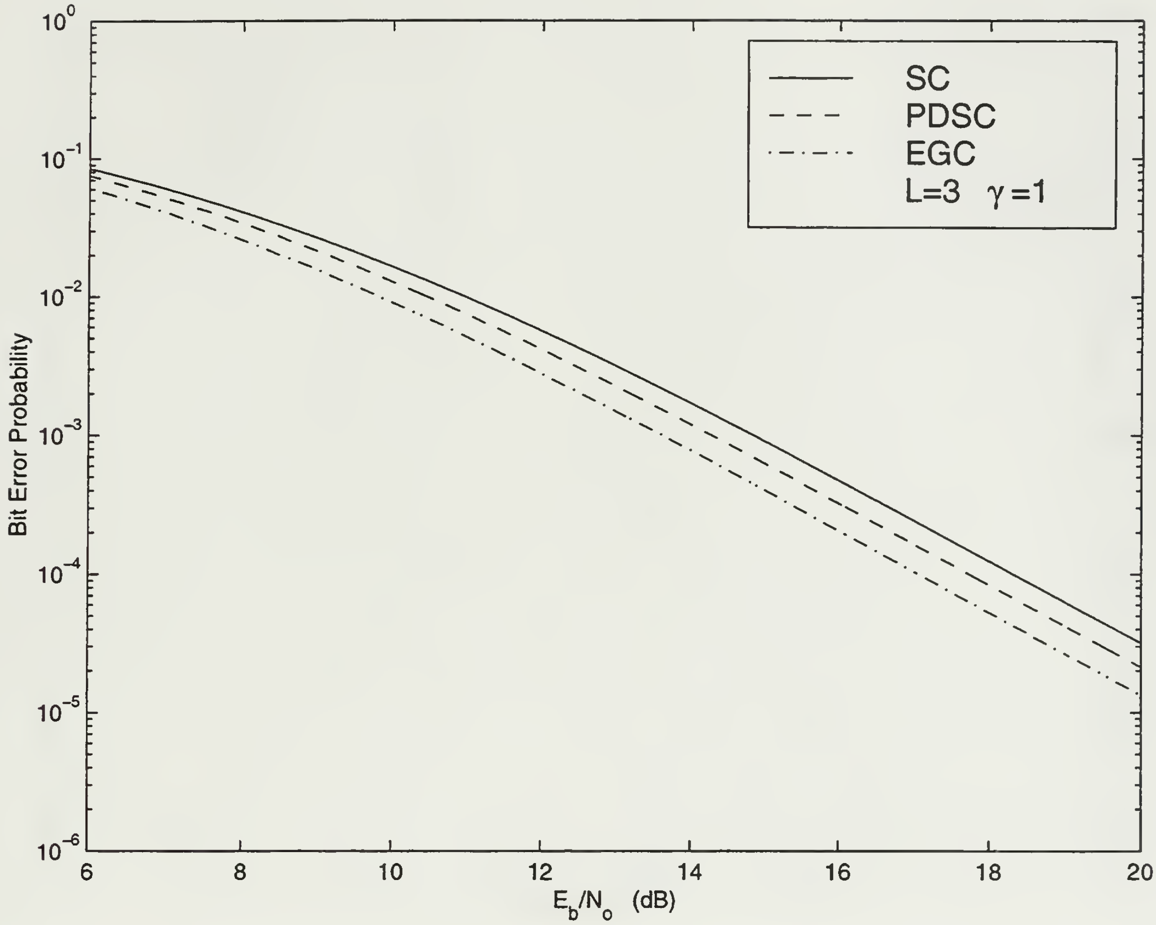

Figure 16. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=3 and $\gamma = 1$.

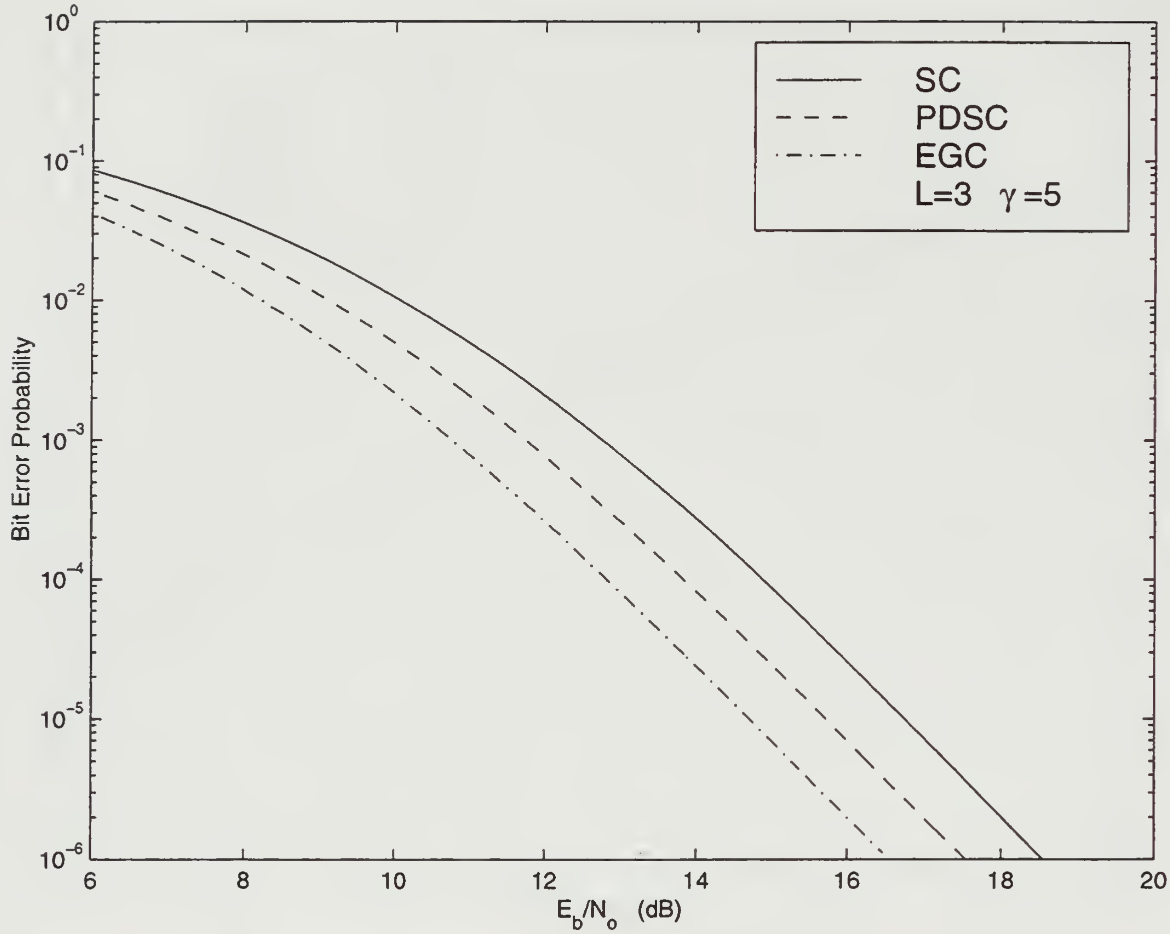

Figure 17. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=3 and $\gamma = 5$.

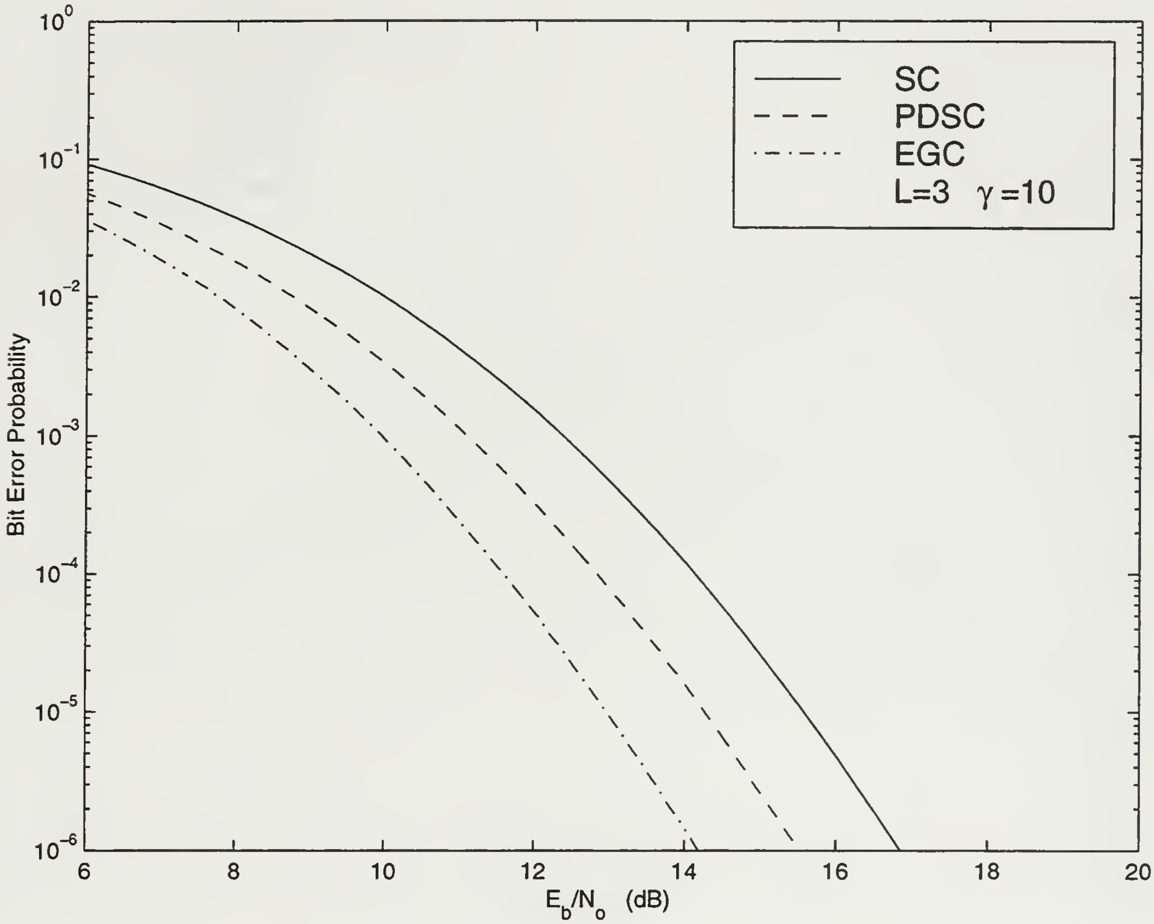

Figure 18. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=3 and $\gamma = 10$.

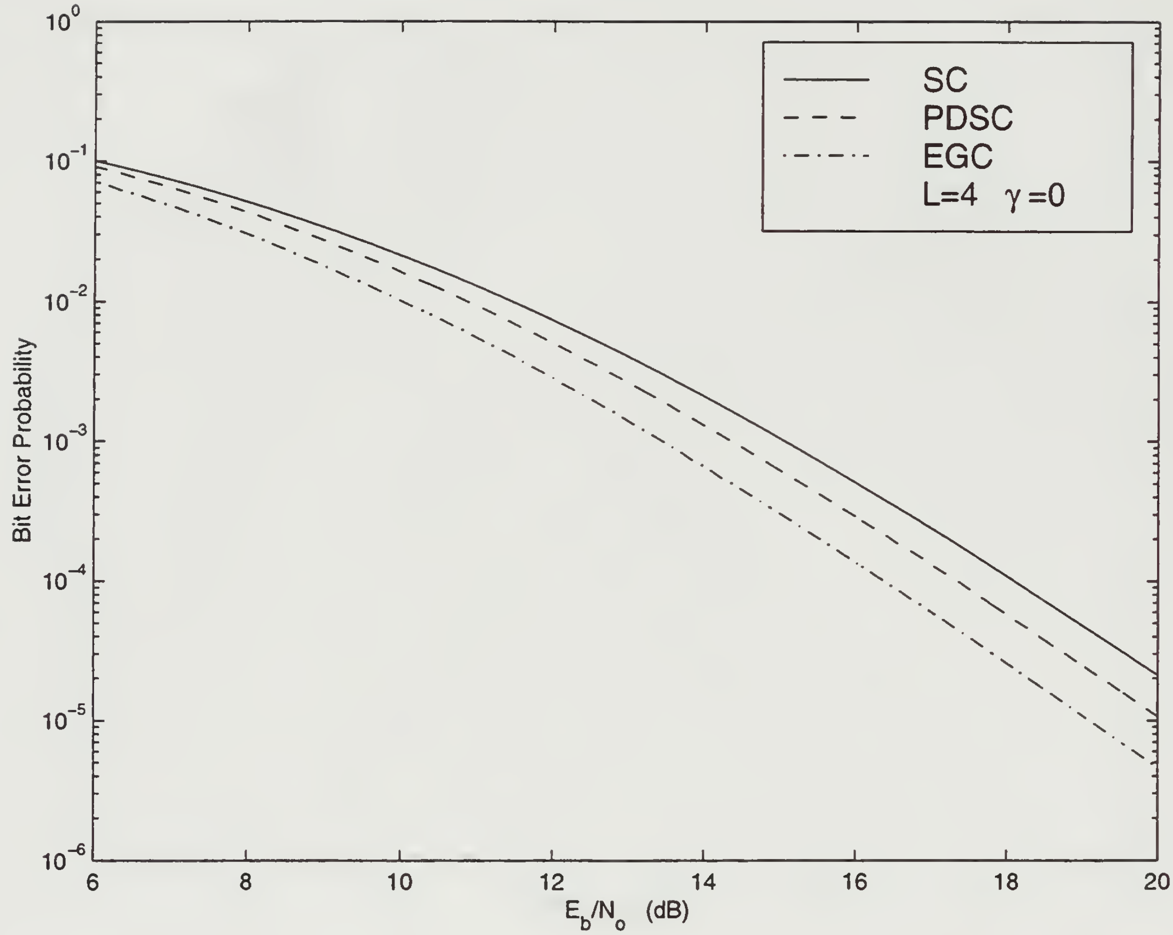

Figure 19. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=4 and $\gamma = 0$.

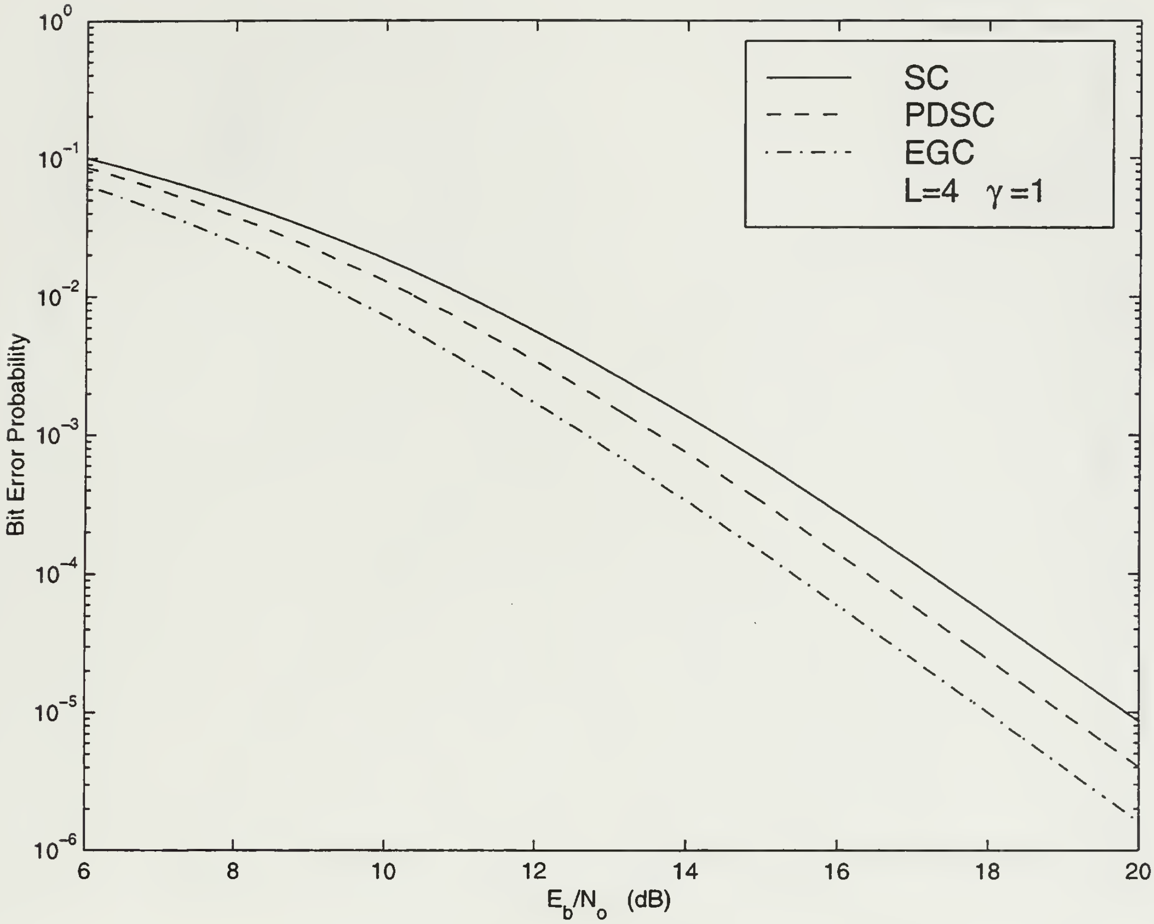

Figure 20. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=4 and $\gamma = 1$.

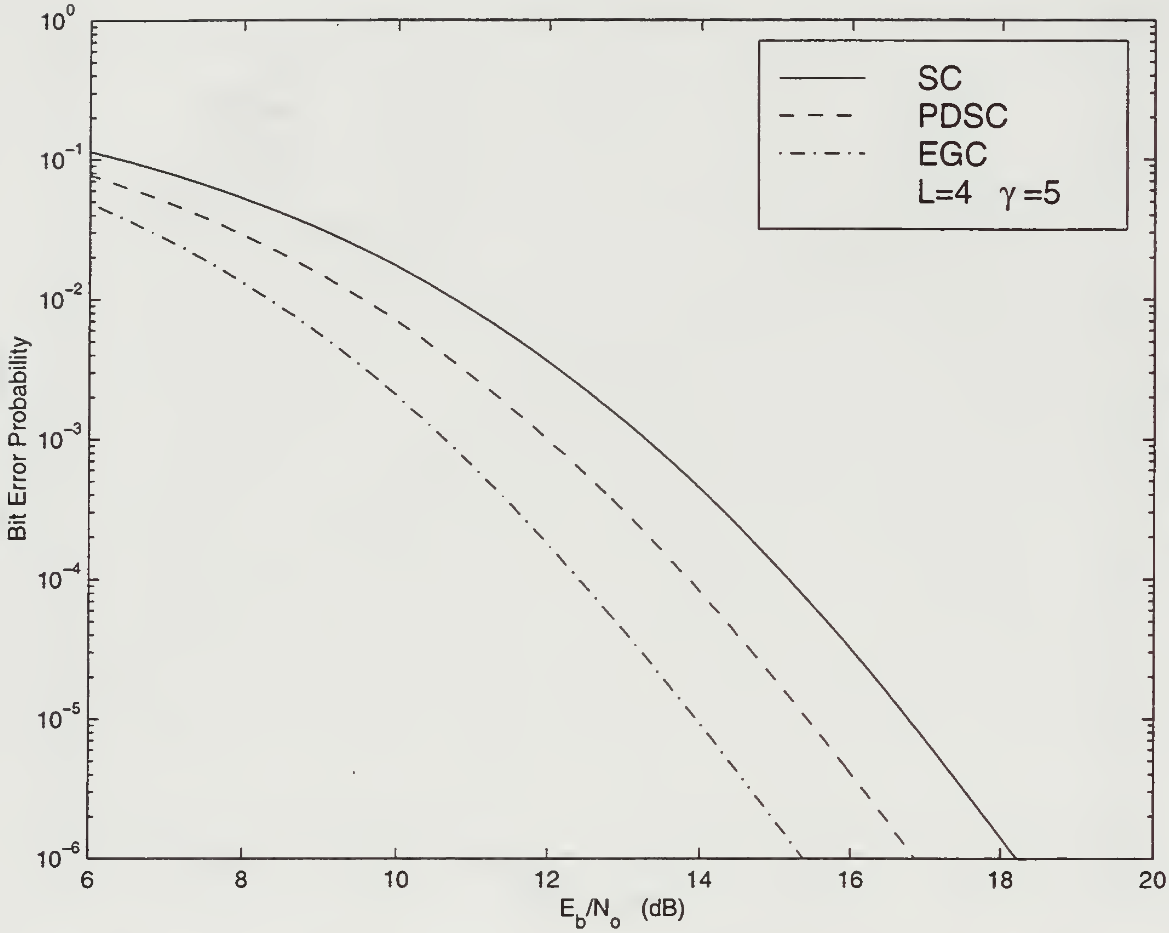

Figure 21. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=4 and $\gamma = 5$.

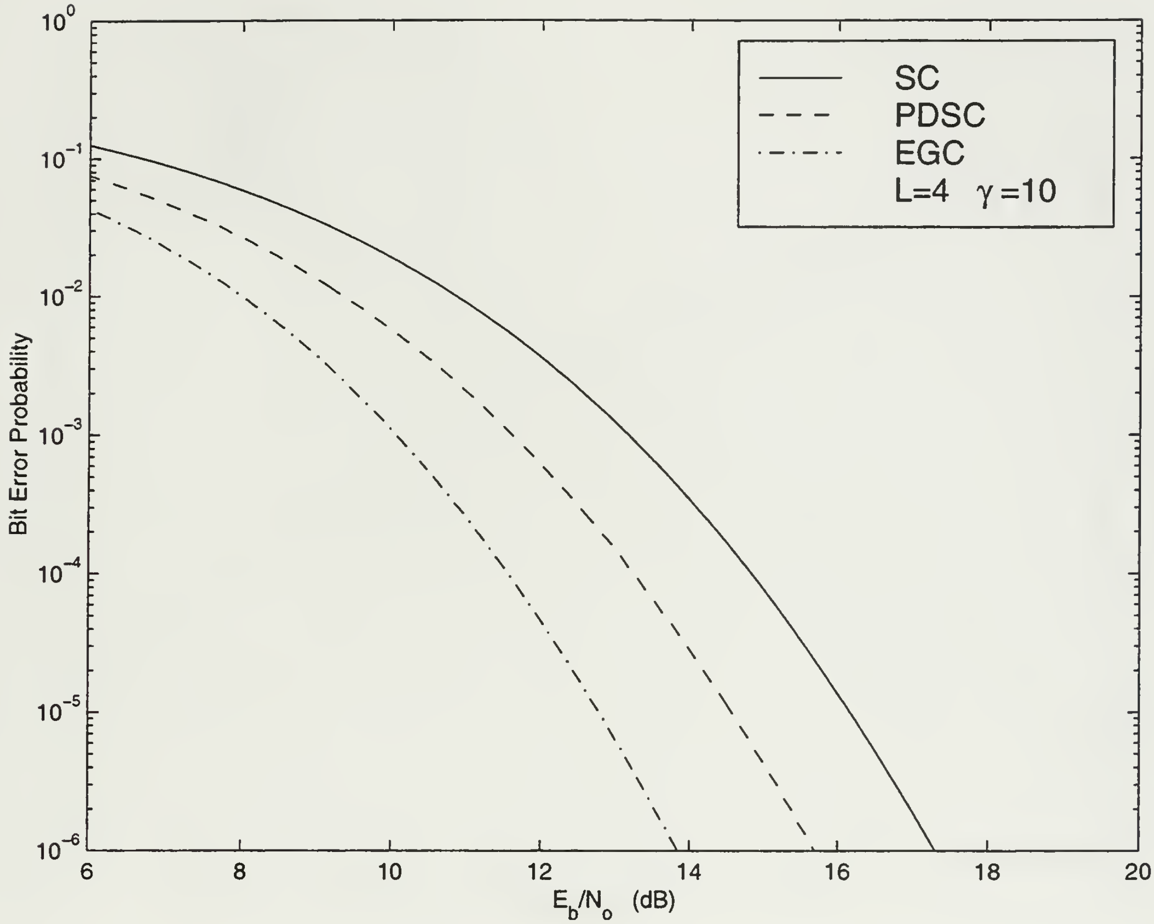

Figure 22. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=4 and $\gamma = 10$.

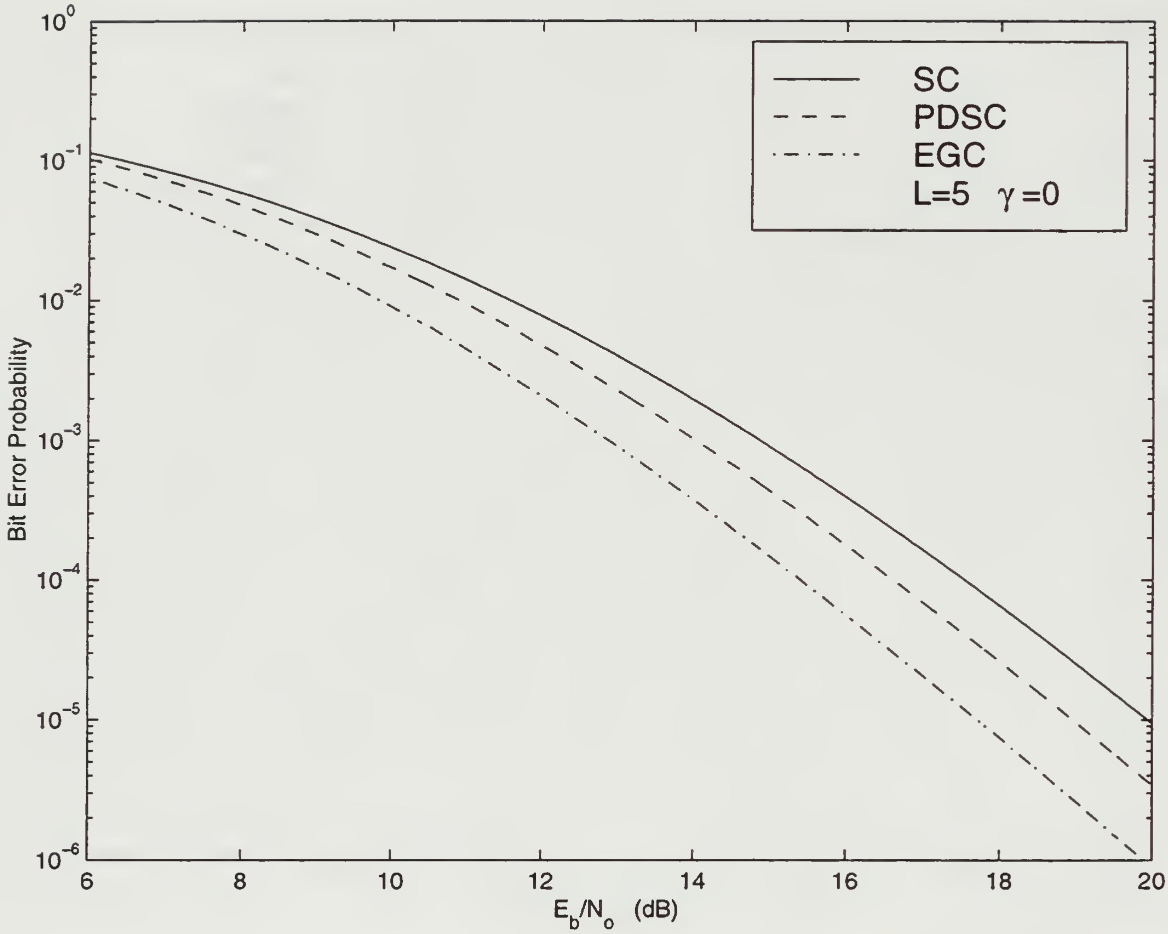

Figure 23. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=5 and $\gamma = 0$.

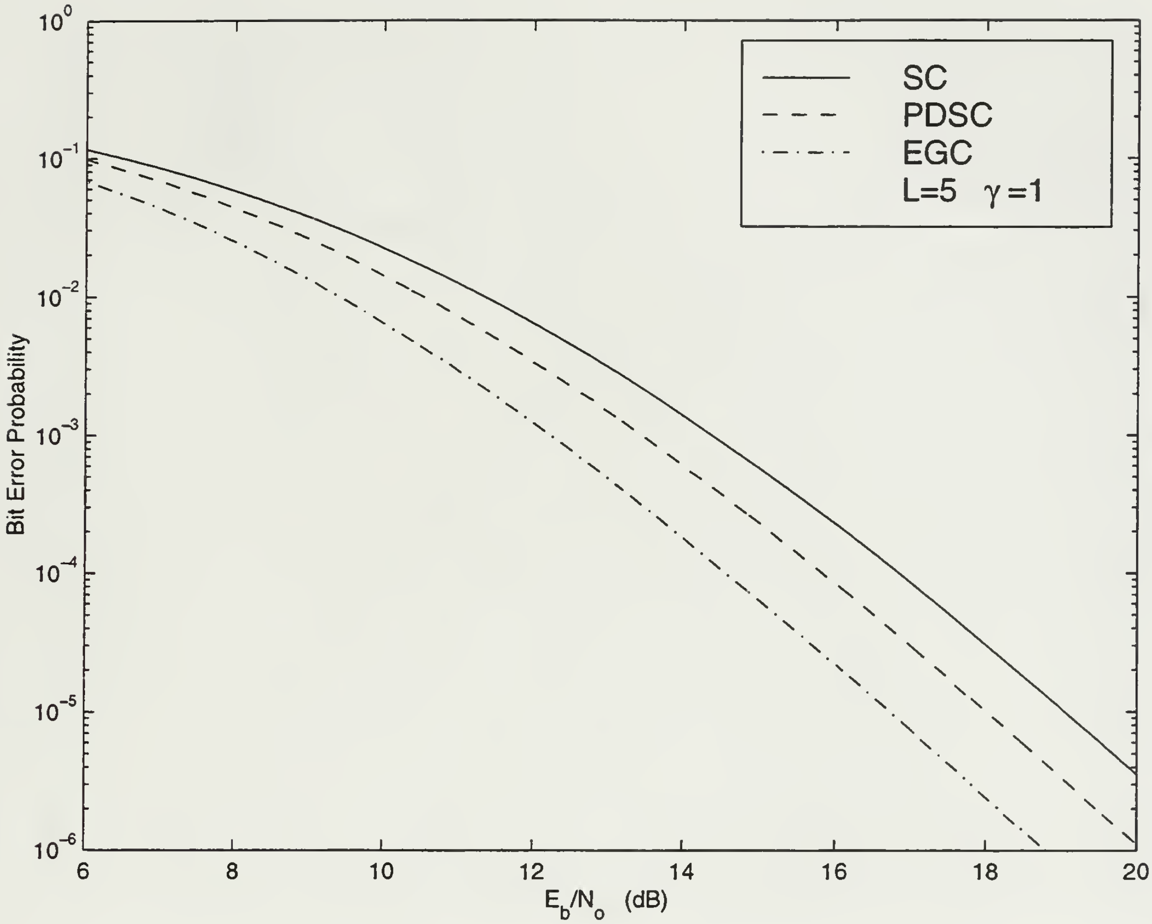

Figure 24. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=5 and $\gamma = 1$.

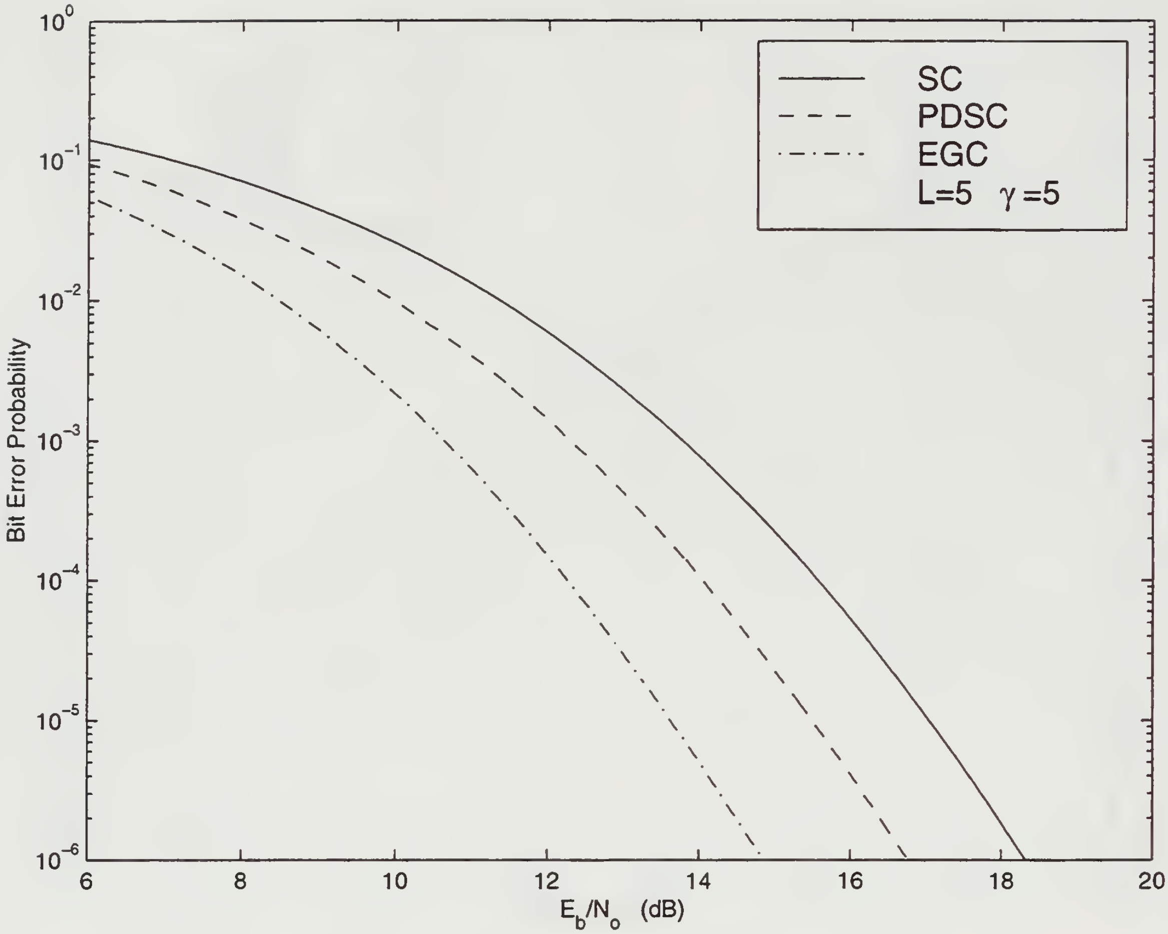

Figure 25. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=5 and $\gamma = 5$.

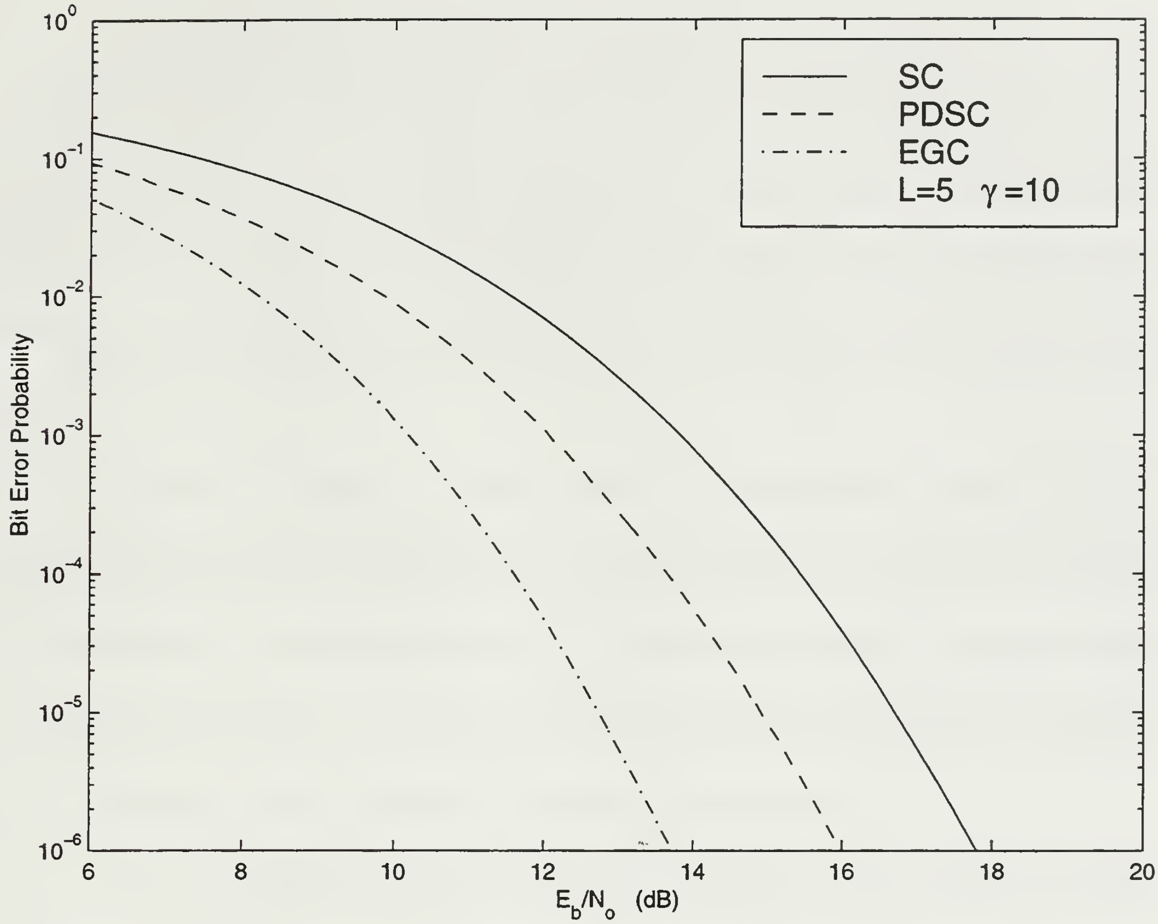

Figure 26. The performance of noncoherent DPSK in a Rician fading channel with SC, PDSC and EGC for L=5 and $\gamma = 10$.

VI. CONCLUSIONS

In this thesis, the EGC, the SC and the PDSC bit error probability performances of a noncoherent DPSK receiver with L^{th} order diversity have been analyzed in a frequency non-selective, slowly fading Rician fading channel. Based on previous analyses of the EGC and the SC techniques for a noncoherent DPSK receiver, it is known that the EGC technique provides a better performance than the SC method in a Rayleigh fading channel [1].

The EGC technique is widely used in communication systems that use noncoherent demodulation, but it has the undesirable features of noncoherent combining loss and receiver complexity which depends on the diversity order L. In a Rician fading channel and at low values of E_b / N_0, the system performance with small L is superior to those with larger L which is due to the noncoherent combining loss.

The SC technique has been suggested as a simpler technique that can provide adequate performance without L dependence. However, it is not an optimal combining technique since it does not use all of the available diversity branches simultaneously. Similar to EGC, performance improvement is observed when E_b / N_0 increases and the same effects of noncoherent combining loss are observed.

Since L may vary with locations as well as time, having receiver complexity dependent on L is undesirable. Therefore, it is of interest to find combining techniques which are independent of L and which can provide adequate performance. The SC

technique is one simple suboptimal design which can achieve acceptable performance under typical channel conditions. The EGC and the SC represent the two extremes in diversity combining with respect to the number of signals used for demodulation.

The PDSC technique requires a less complex receiver than the EGC and may be implemented regardless of the number of the diversity branches. Furthermore it does not require a predetection combiner. As seen from the numerical results, its performance is worse than the EGC but it offers a performance improvement over SC.

APPENDIX: DERIVATION OF THE PROBABILITY DENSITY FUNCTION FOR THE LARGEST RANDOM VARIABLE

In this appendix, the probability density function $f_Z(z)$ of the largest random variable Z is derived. Lets consider L independent, identically distributed (i.i.d.) random variables $X_1, X_2, X_3, ..., X_L$ with the probability density functions

$$f_{X_1}(x_1) = f_{X_2}(x_2) = f_{X_3}(x_3) = ... = f_{X_L}(x_L) = f_X(x) \tag{A.1}$$

and we want to select the largest random variable Z

$$Z = \max\{X_1, X_2, X_3, ... X_L\}. \tag{A.2}$$

According to the order statistics, L random samples are arranged in an increasing order as

$$X_{r_1}(\xi) \leq X_{r_2}(\xi) \leq X_{r_3}(\xi) \leq ... \leq X_{r_k}(\xi) \leq ... \leq X_{r_L}(\xi)\ . \tag{A.3}$$

We next form the L random variables z_i such that

$$z_1 = x_{r_1} \leq z_2 = x_{r_2} \leq ... \leq z_k = x_{r_k} \leq ... \leq z_L = x_{r_L}\ . \tag{A.4}$$

The k^{th} largest random variable between L random variables is called the k^{th} order statistics and we determine its density by using

$$f_k(z)dz = P\{z < z_k \leq z + dz\}\ . \tag{A.5}$$

The event $A = \{z < z_k \leq z + dz\}$ occurs if and only if k-1 of the random variables are less than z and one is in the interval (z, z+dz). By defining the sets as

$$\beta_1 = \{x \leq z\}, \qquad \beta_2 = \{z < x \leq z + dz\} \text{ and } \qquad \beta_3 = \{z + dz < x\} \tag{A.6}$$

the probabilities of events are equal to

$$P(\beta_1) = p_1 = F_x(z), \tag{A.7}$$

$$P(\beta_2) = p_2 = f_x(z)dz \tag{A.8}$$

and

$$P(\beta_3) = p_3 = 1 - F_x(z + \Delta z). \tag{A.9}$$

where $F_X(x)$ is the cumulative distribution and $f_X(x)$ is the density of x. We can define "β_i occurs k_i times" as [6]

$$\frac{L!}{k_1!k_2!k_3!} p_1^{k_1} p_2^{k_2} p_3^{k_3} \quad , \quad k_1 + k_2 + k_3 = L. \tag{A.10}$$

The event A occurs if β_1 occurs k-1 times, β_2 occurs once, and β_3 occurs L-k times. So

$$k_1 = k - 1, \qquad k_2 = 1, \qquad k_3 = L - k \tag{A.11}$$

and by inserting these expressions into (10), we get

$$f_k(z)dz = P\{z < z_k \le z + dz\} = \frac{L!}{(k-1)!(L-k)!} F_x^{k-1}(z) f_x(z)dz\,[1 - F_x(z + dz)]^{L-k}. \tag{A.12}$$

As a result, the k^{th} order statistics is obtained as

$$f_k(z) = \frac{L!}{(k-1)!(L-k)!} F_x^{k-1}(z)[1 - F_x(z)]^{L-k} f_x(z). \tag{A.13}$$

In our case , we want the largest value which means that k=L. So the probability density function of the largest random variable between L random variables is defined by

$$f_Z(z) = L f_X(z) F_x^{L-1}(z). \tag{A.14}$$

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